

(1) [8 points] Let $f(x) = x^3 + 2x^2$ and $g(x) = 3x^2 - 1$.

(i) Determine fg and state the domain.

$$\begin{aligned}(fg)(x) &= f(x)g(x) = (x^3 + 2x^2)(3x^2 - 1) \\ &= 3x^5 + 6x^4 - x^3 - 2x^2\end{aligned}$$

Domain of fg is the set of real x common to domains of f & g , which is $(-\infty, \infty)$.

(ii) Determine $\frac{f}{g}$ and state the domain.

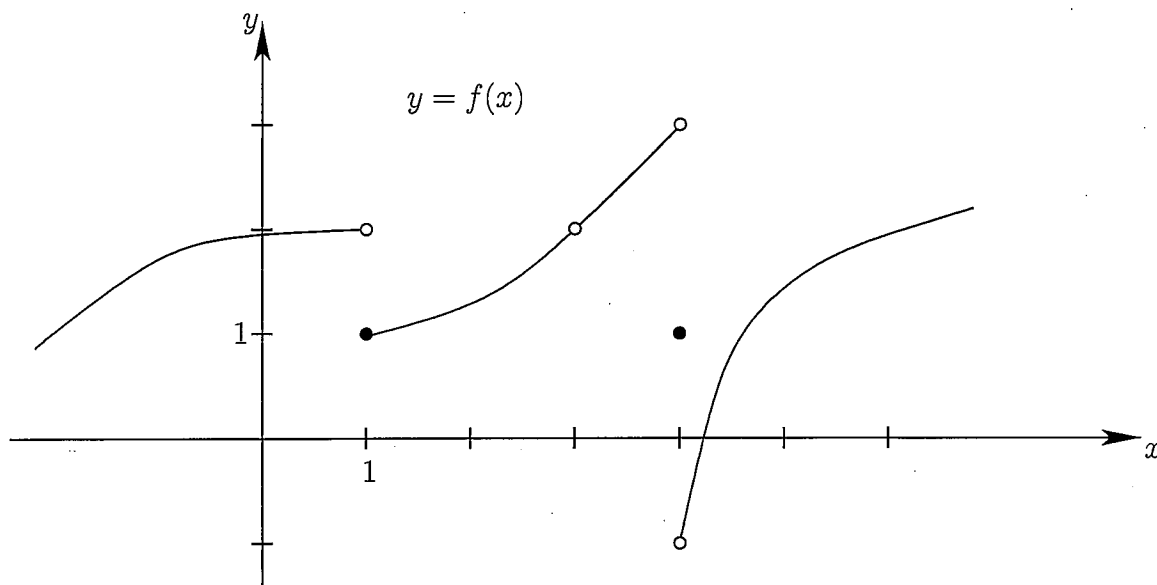
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^3 + 2x^2}{3x^2 - 1}$$

Domain of $\frac{f}{g}$ is the set of real x common to domains of f & g but excluding x for which $g(x) = 0$.

$$g(x) = 0 \Rightarrow 3x^2 - 1 = 0 \Rightarrow x = \pm\sqrt{\frac{1}{3}}$$

\therefore Domain of $\frac{f}{g}$ is $(-\infty, -\sqrt{\frac{1}{3}}) \cup (-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}) \cup (\sqrt{\frac{1}{3}}, \infty)$.

(2) [7 points] Consider the graph of $y = f(x)$:



(i) What is $\lim_{x \rightarrow 1^+} f(x)$?

$$\lim_{x \rightarrow 1^+} f(x) = 1$$

(ii) What is $\lim_{x \rightarrow 1^-} f(x)$?

$$\lim_{x \rightarrow 1^-} f(x) = 2$$

(iii) What is $\lim_{x \rightarrow 3} f(x)$?

$$\lim_{x \rightarrow 3} f(x) = 2$$

(iv) What is $\lim_{x \rightarrow 4} f(x)$?

$$\lim_{x \rightarrow 4} f(x) \text{ does not exist}$$

$$\left(\text{since } \lim_{x \rightarrow 4^-} f(x) \neq \lim_{x \rightarrow 4^+} f(x) \right)$$