Math 152 Sec S0601/S0602
Final Exam Apr 19 2006 Name:

Question 1: **[40 points]** This question consists of 20 short answer problems each worth 2%. For each problem, clearly write your answer in the box to the right AS ONLY THAT ANSWER WILL BE GRADED. The solution to each problem is short, requiring no more space than that given. Although no part marks are awarded, show your work clearly in case it is needed to support your final answer.

(a) Complete the square for $y = x^2 - 8x + 12$.

$y = (x-4)^2 - 16 + 12$

$y = (x-4)^2 - 4$

$\boxed{y = (x-4)^2 - 4}$

(b) Let $f(x) = 2x^2 + 1$. Compute and simplify $\dfrac{f(x+h) - f(x)}{h}$.

$\dfrac{2(x+h)^2 + 1 - 2x^2 - 1}{h}$

$= \dfrac{\cancel{2x^2} + 4xh + 2h^2 \cancel{+1} \cancel{-2x^2} \cancel{-1}}{h}$

$= 4x + 2h$

$\boxed{4x + 2h}$

(c) Simplify $\dfrac{(-2)^3(x^2y)^2 - 4x^2yz}{-16(xy)^3z^2}$.

$= \dfrac{-\cancel{8}^{2} x^{\cancel{4}\,2} y^{\cancel{2}\,1} \cancel{-4}^{1} \cancel{x^2} \cancel{y} z}{4\cancel{-16}\, x^{\cancel{3}\,1} y^{\cancel{3}\,2} z^2}$

$= \dfrac{2x^2y - z}{4xy^2z^2}$

$\boxed{\dfrac{2x^2y + z}{4xy^2z^2}}$

(d) If $ax^2 + 6x + 9 = 0$ has only one solution, what is a?

$b^2 - 4a \cdot 9 = 0$ (distinct)

$36 - 36a = 0$

$a = 1$

$\boxed{a = 1}$

or $a = 0$

(e) Convert 15° to radians.

$15^\circ \times \frac{\pi \text{ rad}}{180^\circ}$

$= \frac{\pi}{12}$ rad

$\boxed{\frac{\pi}{12}}$

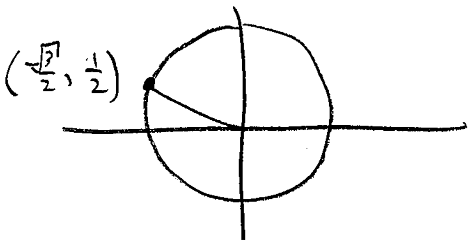
(f) Convert $-\frac{11\pi}{6}$ to degrees.

$-\frac{11\cancel{\pi}}{\cancel{6}}$ rad $\cdot \frac{\overset{30}{\cancel{180^\circ}}}{\cancel{\pi} \text{ rad}}$

$= -330^\circ$

$\boxed{-330^\circ}$

(g) Find $\sin(-7\pi/6)$ exactly.



$$\boxed{\frac{1}{2}}$$

(h) Find $\cos(-37\pi/6)$ exactly.

$= \cos\left(\frac{37\pi}{6}\right)$

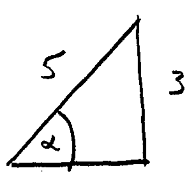
$= \cos\left(\frac{36\pi + \pi}{6}\right)$

$= \cos\left(6\pi + \frac{\pi}{6}\right)$

$= \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$$\boxed{\frac{\sqrt{3}}{2}}$$

(i) If α is an angle in a right triangle with $\csc\alpha = 5/3$, what is $\cos\alpha$?



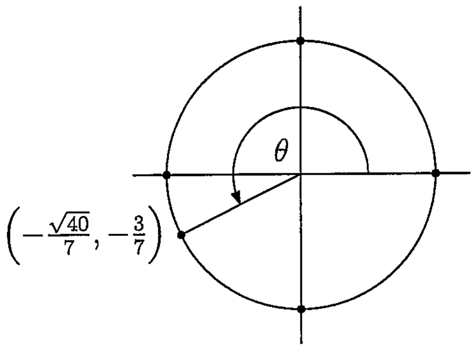
$\therefore 4$

$\cos(\alpha) = \frac{4}{5}$

$$\boxed{\frac{4}{5}}$$

(j) Using the figure below, find $\cot\theta$.

$$\boxed{\frac{\sqrt{40}}{3}.}$$

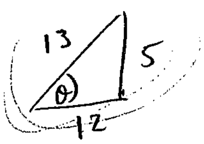


$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{-\sqrt{40}}{7}}{-\frac{3}{7}} = \frac{\sqrt{40}}{3}.$$

(k) If $\sin\theta = 5/13$ where $0 \le \theta \le \pi/2$, find $\sin(\theta - \pi/4)$.

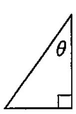
$$\sin\left(\theta - \frac{\pi}{4}\right) = \sin\theta\cos\left(\frac{\pi}{4}\right) - \cos(\theta)\sin\left(\frac{\pi}{4}\right)$$

$$\boxed{\frac{-7\sqrt{2}}{26} \text{ or } \frac{-7}{13\sqrt{2}}}$$



$$= \frac{5}{13}\cdot\frac{1}{\sqrt{2}} - \frac{12}{13}\frac{1}{\sqrt{2}}$$

$$= \frac{-7}{13\sqrt{2}} = \frac{-7\sqrt{2}}{26}$$

(l) If $\tan\theta = a$ in  find an expression for $\cos\theta$ in terms of a.

$$\cos\theta = \frac{1}{\sqrt{1+a^2}}$$

$$\boxed{\frac{1}{\sqrt{1+a^2}}}$$

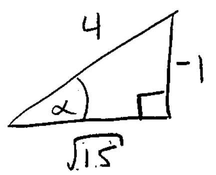
(m) What is the phase shift of $f(x) = -2\sin\left(\frac{2}{3}x - \frac{\pi}{4}\right) - \frac{1}{2}$?

$$f(x) = -2\sin\left[\frac{2}{3}\left(x - \frac{\pi}{4}\cdot\frac{3}{2}\right)\right] - \frac{1}{2}$$

$$= -2\sin\left[\frac{2}{3}\left(x - \frac{3\pi}{8}\right)\right] - \frac{1}{2}$$

$\boxed{\frac{3\pi}{8}}$

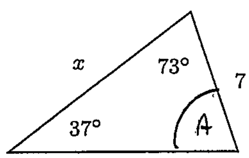
(n) Determine exactly $\tan[\underbrace{\sin^{-1}(-1/4)}_{\alpha}]$.



$\therefore \tan\alpha = \frac{-1}{\sqrt{15}}$

$\boxed{\frac{-1}{\sqrt{15}}}$

(o) Determine x in the following figure (round your answer to 1 decimal place):



$\boxed{10.9}$

$A = 180 - 73 - 37 = 70°$.

$$\frac{\sin(70°)}{x} = \frac{\sin(37°)}{7}$$

$$\therefore \quad x = \frac{7\sin(70°)}{\sin(37°)} \doteq 10.9$$

(p) Find the sum of the arithmetic series $11 + \frac{9}{5} + \cdots + (-173)$.

$a = 11$

$d = \frac{9}{5} - 11 = \frac{-46}{5}$

$\boxed{-1701}$

$-173 = 11 + (n-1)\left(-\frac{46}{5}\right)$

$\therefore \quad n = (-173-11)\left(\frac{-5}{46}\right) + 1 = 21$

$\therefore \quad S_{21} = \frac{21}{2}\left(11 - 173\right) = -1701$

(q) A geometric series has $a_7 = -\frac{3}{128}$ and $\frac{a_9}{a_{14}} = -32$. What is a_1?

$\frac{\cancel{a} r^8}{\cancel{a} r^{13}} = \frac{1}{r^5} = -32$

$\boxed{a_1 = -\frac{3}{2}}$

$\therefore \ r = \left(\frac{-1}{32}\right)^{\frac{1}{5}} = \frac{-1}{2}$

$\therefore \quad a_1\left(-\frac{1}{2}\right)^6 = -\frac{3}{128}$

$a_1 \frac{1}{\cancel{64}} = \frac{-3}{{}_2\cancel{128}} \qquad \therefore \quad a_1 = \frac{-3}{2}$

(r) Let $\mathbf{A} = \begin{bmatrix} -2 & 3 \\ 1 & -4 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} -1 & 3 \\ -2 & 0 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$. Compute $(2\mathbf{A} - \mathbf{B})\mathbf{C}$.

$\boxed{\begin{bmatrix} 21 & -9 \\ -40 & 16 \end{bmatrix}}$

$2A = \begin{bmatrix} -4 & 6 \\ 2 & -8 \end{bmatrix}$

$2A - B = \begin{bmatrix} -3 & 3 \\ 4 & -8 \end{bmatrix}$

$(2A-B)C = \begin{bmatrix} -3 & 3 \\ 4 & -8 \end{bmatrix}\begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 21 & -9 \\ -40 & 16 \end{bmatrix}$

(s) If **A** is size 3×5 and **C** is size 2×7, what size is **B** in order for **ABC** to be defined?

$\boxed{5 \times 2}$

(t) Solve $\mathbf{A}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ for x and y given that $\mathbf{A}^{-1} = \begin{bmatrix} 2 & 1 \\ -5/2 & 3/2 \end{bmatrix}$.

$x = 1$
$y = -\frac{19}{2}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -\frac{5}{2} & \frac{3}{2} \end{bmatrix}\begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -\frac{19}{2} \end{bmatrix}$$

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Question 2: [10 points]
Write the equation of the circle

$$2x^2 - 2x + 2y^2 + \frac{16}{3}y = \frac{251}{18}$$

in the form $(x-a)^2 + (y-b)^2 = r^2$. State the centre and radius of the circle.

$$x^2 - x + y^2 + \frac{8}{3}y = \frac{251}{36}$$

$$\left(x-\frac{1}{2}\right)^2 - \frac{1}{4} + \left(y+\frac{4}{3}\right)^2 - \frac{16}{9} = \frac{251}{36}$$

$$\left(x-\frac{1}{2}\right)^2 + \left(y+\frac{4}{3}\right)^2 = \frac{251}{36} + \frac{1}{4} + \frac{16}{9}$$

$$= \frac{251+9+64}{36}$$

$$= \frac{324}{36}$$

$$= 3^2$$

$$\therefore \left(x-\frac{1}{2}\right)^2 + \left(y+\frac{4}{3}\right)^2 = 3^2$$

centre $\left(\frac{1}{2}, \frac{-4}{3}\right)$

radius 3.

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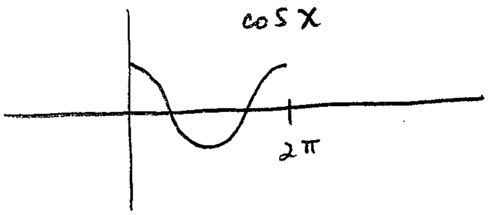
Question 3: [10 points]
Carefully graph the function

$$f(x) = -2\cos\left(\pi x + \frac{\pi}{2}\right) + 1\,.$$

State the period, phase shift and amplitude.

$$f(x) = -2\cos\left[\pi\left(x+\tfrac{1}{2}\right)\right] + 1$$

1



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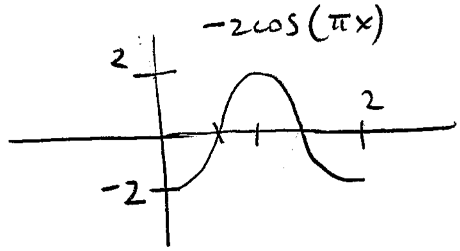

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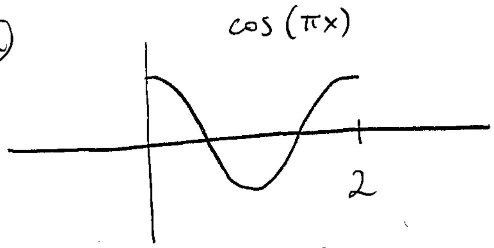

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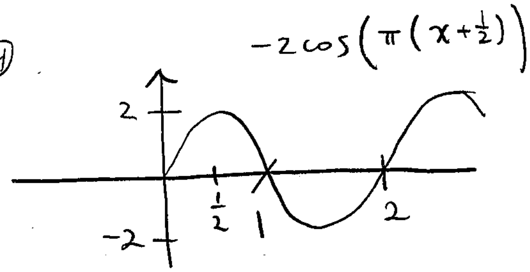

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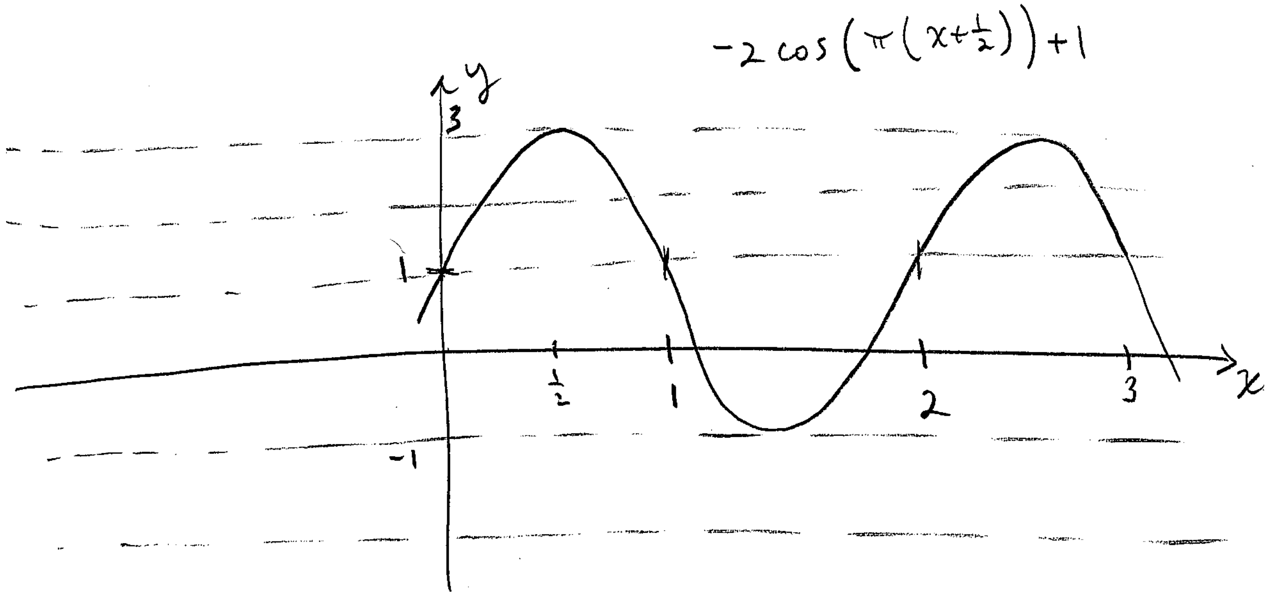
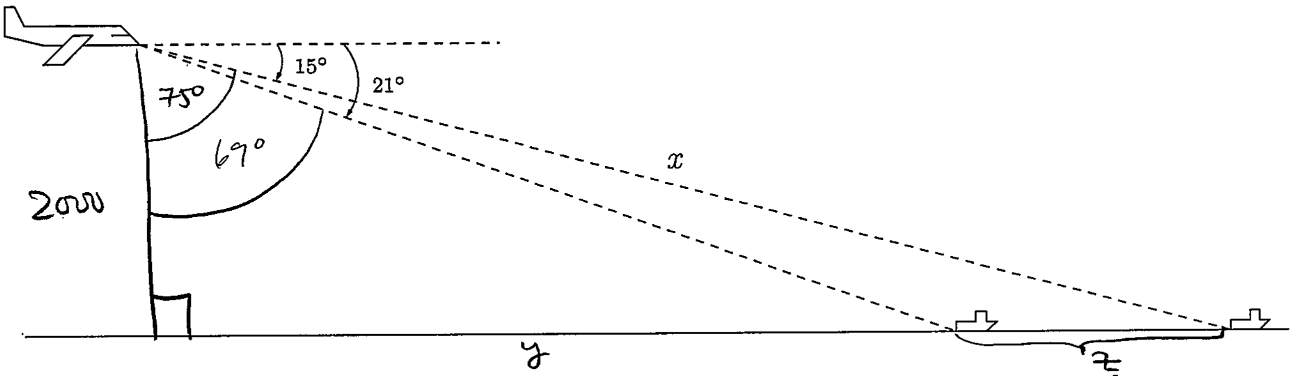


Period $\frac{2\pi}{\pi} = 2$

phase shift $= -\frac{1}{2}$

amplitude $= 2$.

Question 4: [10 points]
A surveillance plane flying at an altitude of 2000 metres spots two suspicious ships in the distance directly in line with its flight path. The angle of depression to the more distant ship is 15°, while that to the closer ship is 21° as shown in the figure below.



(a) [6 points] Find the distance between the two ships. Round your answer to the nearest metre.

$$\tan(69^\circ) = \frac{y}{2000}$$

$$\therefore y = 2000\tan(69^\circ)$$

$$\tan(75^\circ) = \frac{y+z}{2000}$$

$$\therefore z = 2000\tan(75^\circ) - y$$

$$= 2000\left[\tan(75^\circ) - \tan(69^\circ)\right]$$

$$\doteq \boxed{2254 \text{ m}}$$

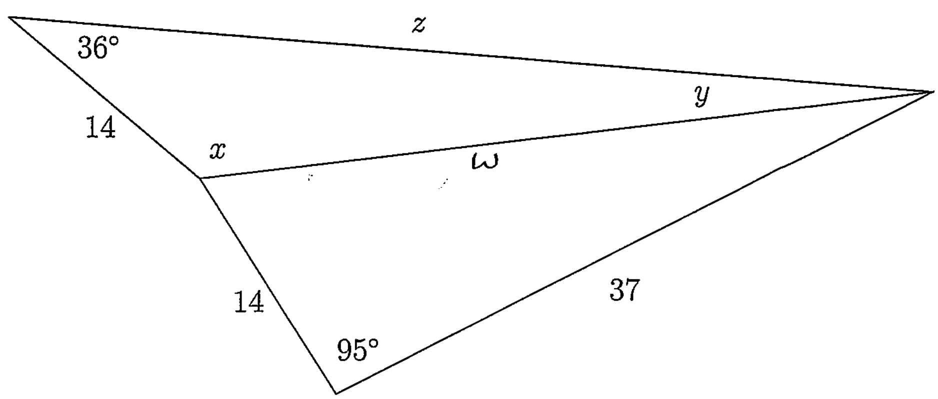
(b) [4 points] What is the distance x between the plane and the more distant ship? Round your answer to the nearest metre.

$$x^2 = 2000^2 + (y+z)^2$$

$$\therefore x = \sqrt{2000^2 + (2000\tan(75^\circ))^2}$$

$$\doteq \boxed{7727 \text{ m}}$$

Question 5: [10 points]
Determine the angles x and y and the side z in the following figure. Round all final answers to one decimal.



$$w = \sqrt{14^2 + 37^2 - 2(14)(37)\cos(95^\circ)} \doteq 40.685$$

$$\left.\frac{\sin 36^\circ}{w} = \frac{\sin y}{14}\right\} \therefore \quad y \doteq \sin^{-1}\left[\frac{14 \sin(36^\circ)}{40.685}\right]$$

$$\boxed{y \doteq 11.7^\circ}$$

$$\therefore \; x = 180 - 36 - y$$

$$\boxed{x \doteq 132.3^\circ}$$

$$z = \sqrt{14^2 + w^2 - 2(14)(w)\cos(x)}$$

$$\boxed{z = 51.2}$$

Question 6: [10 points]
Let

$$\mathbf{A} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

(a) **[5 points]** Find $\mathbf{A}^{-1}$.

(b) **[5 points]** Solve the following system of equations

$$\begin{aligned} 3x - 2y - 4z &= -3 \\ 3x - 2y - 5z &= 2 \\ -x + y + 2z &= -2 \end{aligned}$$

(Your result from part (a) should make this easy.)

$$\left[\begin{array}{ccc|ccc} 3 & -2 & -4 & 1 & 0 & 0 \\ 3 & -2 & -5 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \leftrightarrow R_3$ $\left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 0 & 0 & 1 \\ 3 & -2 & -5 & 0 & 1 & 0 \\ 3 & -2 & -4 & 1 & 0 & 0 \end{array}\right]$

$(-1)R_1$ $\left[\begin{array}{ccc|ccc} 1 & -1 & -2 & 0 & 0 & -1 \\ 3 & -2 & -5 & 0 & 1 & 0 \\ 3 & -2 & -4 & 1 & 0 & 0 \end{array}\right]$

$(-3)R_1 + R_2$
$(-3)R_1 + R_3$ $\left[\begin{array}{ccc|ccc} 1 & -1 & -2 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 1 & 0 & 3 \end{array}\right]$.

$R_2 + R_1$
$(-1)R_2 + R_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 0 & 1 & 3 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{array}\right]$

R_1
$(-1)R_3 + R_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & -1 & 2 & 3 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{array}\right]$.

(a) $\therefore A^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix}$.

(b) $\begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} -7 \\ 1 \\ -5 \end{bmatrix}$

Question 7: [10 points]

(a) [**5 points**] Determine the coefficient of x^2 in the expansion of

$$\left(\sqrt{x}+\frac{3}{\sqrt{x}}\right)^8.$$

$$\binom{8}{k}\left(x^{\frac{1}{2}}\right)^{8-k}\cdot\left(3x^{-\frac{1}{2}}\right)^{k} = Cx^2$$

$$\binom{8}{k}3^{k}x^{4-\frac{k}{2}}\cdot x^{-\frac{k}{2}} = Cx^2$$

$$\therefore\ 4-k=2$$
$$k=2$$

$$\therefore \text{Coefficient } \binom{8}{2}3^2 = \frac{8!}{2!\,6!}\cdot 3^2 = \frac{\cancel{8}^{4}\cdot 7\cdot 3\cdot 3}{\cancel{2}} = \boxed{252}$$

(b) [**5 points**] Recall that P dollars invested at an interest rate of $i\%$ compounded annually accumulates to $P(1+i/100)^n$ dollars at the end of n years. Suppose \$1500 is deposited at the beginning of each year for 30 years into an investment paying 8% compounded annually. What is the total value of the investment at the end of the 30 years? (Round your answer to the nearest dollar.)

$$1500(1.08)^{30}+1500(1.08)^{29}+\cdots+1500(1.08)$$

$$= 1500(1.08)\left(\frac{1-1.08^{30}}{1-1.08}\right)$$

$$\doteq \boxed{\$183,519}$$