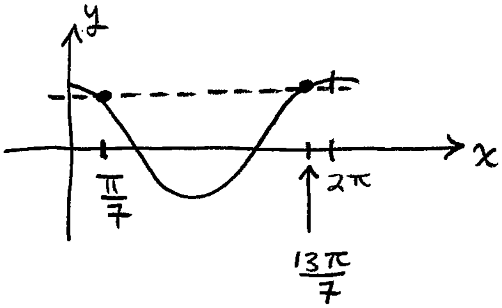
Question 1:

(a) [**5 points**] Find the exact value of $\cos^{-1}\left[\cos\left(\frac{13\pi}{7}\right)\right]$.



$\cos\left(\frac{13\pi}{7}\right) = \cos\left(\frac{\pi}{7}\right)$

$\therefore \cos^{-1}\left[\cos\left(\frac{13\pi}{7}\right)\right] = \cos^{-1}\left[\cos\left(\frac{\pi}{7}\right)\right]$

$= \boxed{\frac{\pi}{7}}$

(b) [**5 points**] Express as an algebraic expression in u: $\sin\left(\cos^{-1} u\right)$. Your answer should not contain any trigonometric functions.

let $\theta = \cos^{-1} u$

$\therefore \cos\theta = u$ where $0 \le \theta \le \pi$; find $\sin\theta$.

$\sin^2\theta + \cos^2\theta = 1$,

$\therefore \sin\theta = \sqrt{1-\cos^2\theta}$ } positive square root since $\sin\theta \ge 0$ for $0 \le \theta \le \pi$.

$= \boxed{\sqrt{1-u^2}}$

Question 2:

(a)[3 points] Determine the exact value of $\cos\left(\frac{7\pi}{12}\right)$. (Hint: $\frac{7}{12} = \frac{3}{12} + \frac{4}{12} = \frac{(7/6)}{2}$.)

$$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$$

$$= \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$$

$$= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} - \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}$$

$$= \boxed{\frac{1-\sqrt{3}}{2\sqrt{2}}}$$

(b)[3 points] Determine the exact value of $\sin\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(-1)\right]$.

$$\sin\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(-1)\right]$$

$$= \sin\left[\frac{\pi}{3} + \pi\right]$$

$$= \sin\left[\frac{4\pi}{3}\right]$$

$$= \boxed{-\frac{\sqrt{3}}{2}}$$

(c)[4 points] Find the exact value of $\sin\left(-\frac{\pi}{8}\right)$.

$$\sin\left(-\frac{\pi}{8}\right) = -\sin\left(\frac{\left(\frac{\pi}{4}\right)}{2}\right)$$

$$= -\sqrt{\frac{1-\cos\left(\frac{\pi}{4}\right)}{2}}$$

$$= -\sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}}$$

$$= \boxed{-\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}}$$

Question 3:

(a) [**5 points**] Solve the following for $0 \leq \theta < 2\pi$: $2\cos^2\theta + 5\cos\theta - 3 = 0$

let $u = \cos\theta$: $2u^2 + 5u - 3 = 0$

$(2u-1)(u+3) = 0$

$2u - 1 = 0$, $u + 3 = 0$

$u = \frac{1}{2}$, $u = -3$

$\therefore \cos\theta = \frac{1}{2}$, $\underbrace{\cos\theta = -3}_{\text{no solutions.}}$

$\therefore \boxed{\theta = \frac{\pi}{3}, \frac{5\pi}{3}}$

(b) [**5 points**] Solve the following for $0 \leq \theta < 2\pi$: $\cos(2\theta) = \sin\theta$

$\cos(2\theta) = \sin\theta$

$1 - 2\sin^2\theta = \sin\theta$

$2\sin^2\theta + \sin\theta - 1 = 0$

let $u = \sin\theta$:

$2u^2 + u - 1 = 0$

$u = \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)}$

$= \frac{-1 \pm 3}{4}$

$= \frac{1}{2}, -1$

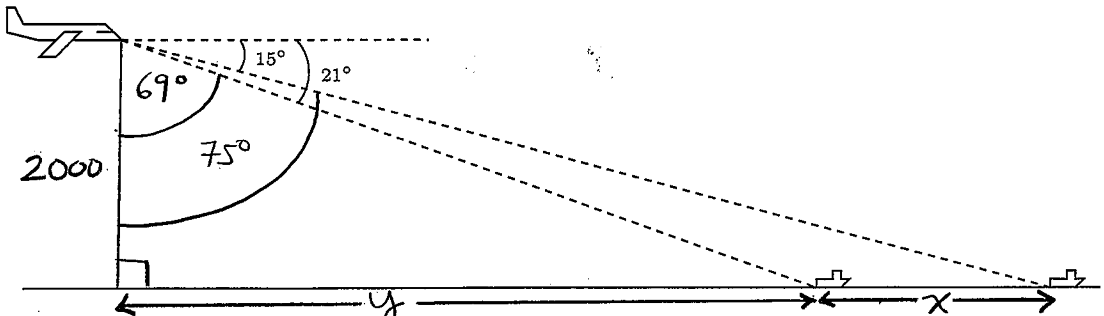
$\therefore \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin\theta = -1 \Rightarrow \theta = \frac{3\pi}{2}$

$\therefore \boxed{\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$

Question 4:

(a) **[5 points]** A surveillance plane flying at an altitude of 2000 metres spots two suspicious ships in the distance directly in line with its flight path. The angle of depression to the more distant ship is 15°, while that to the closer ship is 21° as shown in the figure below.



Find the distance between the two ships. Round your answer to the nearest metre.

$$\tan(75°) = \frac{x+y}{2000}$$

$$\therefore \quad x = 2000\tan(75°) - y$$

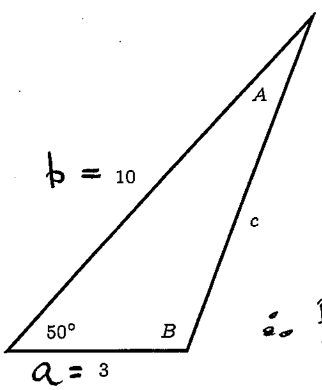
$$\tan(69°) = \frac{y}{2000}$$

$$\therefore \quad y = 2000\tan(69°)$$

$$\therefore \quad x = 2000\tan(75°) - 2000\tan(69°)$$

$$\boxed{x \approx 2254 \text{ m}}$$

(b) **[5 points]** Solve for all missing sides and angles in the following triangle. Round final answers to one decimal place.



$$c^2 = 10^2 + 3^2 - 2(10)(3)\cos(50°)$$

$$\therefore \quad c = \sqrt{100 + 9 - 60\cos(50°)} \approx 8.3924$$

$$b^2 = a^2 + c^2 - 2ac\cos(B)$$

$$\therefore \quad B = \cos^{-1}\left[\frac{a^2 + c^2 - b^2}{2ac}\right]$$

$$= \cos^{-1}\left[\frac{3^2 + 8.3924^2 - 10^2}{(2)(3)(8.3924)}\right] \approx 114.1080°$$

$$\therefore \quad A = 180 - 50 - 114.1080 \approx 15.892°$$

$$\boxed{\therefore \; c \approx 8.4, \; B \approx 114.1°, \; A \approx 15.9°}$$

Question 5 [10 points]: Solve the following system of equations **using matrix reduction** (no credit will be given for using any other method). Clearly state the row operations used at each step and clearly state the solution set.

$$\begin{aligned} 2x - 3y + 4z &= -15 \\ x - y + z &= -4 \\ 5x + y - 2z &= 12 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & -3 & 4 & -15 \\ 1 & -1 & 1 & -4 \\ 5 & 1 & -2 & 12 \end{array}\right]$$

$r_1 \leftrightarrow r_2$: $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 2 & -3 & 4 & -15 \\ 5 & 1 & -2 & 12 \end{array}\right]$

$R_2 = r_2 - 2r_1$:
$R_3 = r_3 - 5r_1$: $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 0 & -1 & 2 & -7 \\ 0 & 6 & -7 & 32 \end{array}\right]$

$R_2 = (-1)r_2$: $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 0 & 1 & -2 & 7 \\ 0 & 6 & -7 & 32 \end{array}\right]$

$R_3 = r_3 - 6r_2$: $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 0 & 1 & -2 & 7 \\ 0 & 0 & 5 & -10 \end{array}\right]$

$R_3 = \frac{1}{5}r_3$: $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 0 & 1 & -2 & 7 \\ 0 & 0 & 1 & -2 \end{array}\right]$

$\therefore z = -2$

$y = 7 + 2z = 7 + 2(-2) = 3$

$x = -4 + y - z = -4 + 3 - (-2) = 1$

$$\boxed{\therefore \{x = 1,\ y = 3,\ z = -2\}}$$