Question 1: After a long day of doing math in the summer you decide to sit down outside and enjoy a cold drink. Your beverage had an initial temperature of 4°C when removed from the refrigerator, and within 15 minutes you observe that the temperature of the drink has reached 10°C. The outside temperature is 30°C.

Recall that Newton's Law of Cooling (and Heating) states that the temperature $u(t)$ of an object at time t is

$$u(t) = T + (u_0 - T)e^{kt}$$

where T is the ambient (surrounding) temperature, u_0 is the initial temperature of the object, and k is a constant.

(a) **[7 points]** Use the information above to determine the temperature of the beverage 27 minutes after it has been removed from the refrigerator. (Round your final answer to the nearest degree.)

$T = 30^\circ$

$u_0 = 4^\circ$

$u(15) = 10^\circ$

$\therefore \quad 10 = 30 + (4-30)e^{15k}$

$\therefore \quad e^{15k} = \frac{10-30}{4-30} = \frac{-20}{-26} = \frac{10}{13}$

$\therefore \quad k = \frac{1}{15}\ln\left(\frac{10}{13}\right)$

$\therefore \quad u(27) = 30 + (4-30)\, e^{\frac{1}{15}\ln\left(\frac{10}{13}\right)\cdot 27} \approx \boxed{14^\circ C.}$

(b) **[3 points]** How long will it take for the beverage to reach a temperature of 25°C? (Round to nearest minute.)

Solve $\quad 25 = 30 + (4-30)\, e^{kt} \quad$ for t

$\therefore \quad e^{kt} = \frac{25-30}{4-30} = \frac{-5}{-26} = \frac{5}{26}$

$\therefore \quad kt = \ln\left(\frac{5}{26}\right)$

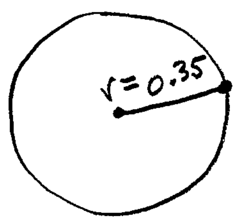
$\therefore \quad t = \frac{\ln\left(\frac{5}{26}\right)}{k} = \frac{\ln\left(\frac{5}{26}\right)}{\frac{1}{15}\ln\left(\frac{10}{13}\right)} \approx \boxed{94 \text{ min.}}$

Question 2:

(a)[2 points] Convert $-17\pi/6$ to degrees.

$$\left(\frac{-17\pi}{6}\right)\left(\frac{180}{\pi}\right) = \boxed{-510^\circ}$$

(b)[4 points] A bicycle wheel has a radius of 0.35 m and rotates at 5 revolutions per second. What is the speed of the bicycle? (Round to one decimal and state units.)



$\omega = 5$ rev. per second

$= (5)(2\pi)$ radians per second

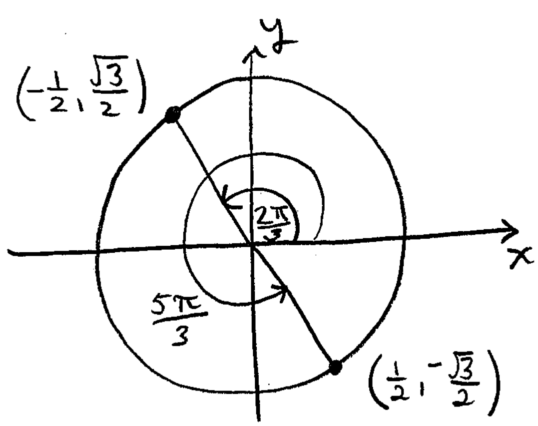
$\therefore$ Speed of bicycle = linear speed of point on wheel

$= \omega r$

$= (5)(2\pi)(0.35)$

$\approx \boxed{11.0 \ \frac{m}{s}}$

(c)[4 points] Find the exact value of $\cos\left(\frac{5\pi}{3}\right) + \sin\left(\frac{2\pi}{3}\right)$.



$\therefore \cos\left(\frac{5\pi}{3}\right) + \sin\left(\frac{2\pi}{3}\right)$

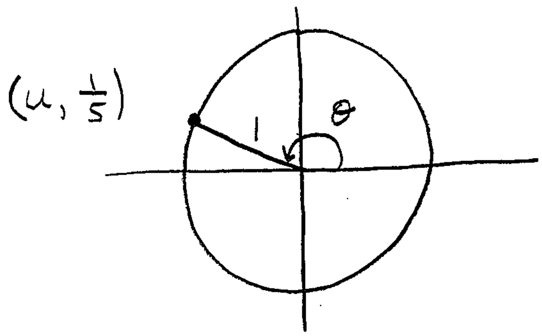
$= \frac{1}{2} + \frac{\sqrt{3}}{2}$

$= \boxed{\frac{1+\sqrt{3}}{2}}$

Question 3:

(a)[4 points] If $\csc\theta = 5$ and $\cos\theta < 0$, determine the value of $\tan\theta$.

$\csc\theta = 5$, so $\sin\theta = \frac{1}{5}$



$u^2 + (\frac{1}{5})^2 = 1$

$\therefore u = -\sqrt{1 - \frac{1}{25}}$

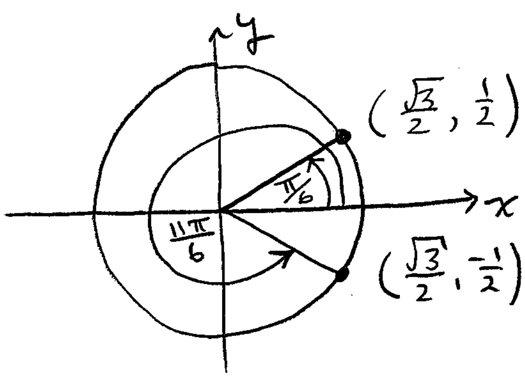
$= -\frac{2\sqrt{6}}{5}$

$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{(\frac{1}{5})}{\frac{-2\sqrt{6}}{5}}$

$= \boxed{\frac{-1}{2\sqrt{6}} \text{ or } \frac{-\sqrt{6}}{12}}$

(b)[3 points] Determine the two angles θ such that $0 \le \theta < 2\pi$ and $\cos\theta = \sqrt{3}/2$.



$\boxed{\therefore \theta = \frac{\pi}{6}, \quad \theta = \frac{11\pi}{6}}$

(c)[3 points] If $\sin\left(\frac{7\pi}{12}\right) = a$, determine $\sin\left(-\frac{31\pi}{12}\right)$.

$\sin\left(-\frac{31\pi}{12}\right) = -\sin\left(\frac{31\pi}{12}\right)$

$= -\sin\left(\frac{24\pi}{12} + \frac{7\pi}{12}\right)$

$= -\sin\left(2\pi + \frac{7\pi}{12}\right)$

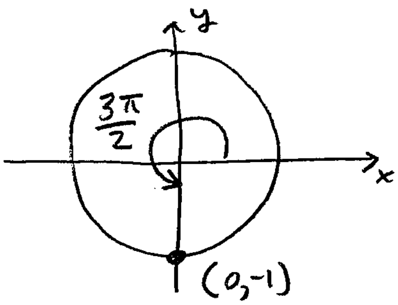
$= -\sin\left(\frac{7\pi}{12}\right)$

$= \boxed{-a}$

Question 4:

(a)[3 points] Determine $\sin\left(\frac{1003\pi}{2}\right)$

$$\sin\left(\frac{1003\pi}{2}\right) = \sin\left(\frac{1000\pi}{2} + \frac{3\pi}{2}\right)$$



$$= \sin\left((250)(2\pi) + \frac{3\pi}{2}\right)$$

$$= \sin\left(\frac{3\pi}{2}\right)$$

$$= \boxed{-1}$$

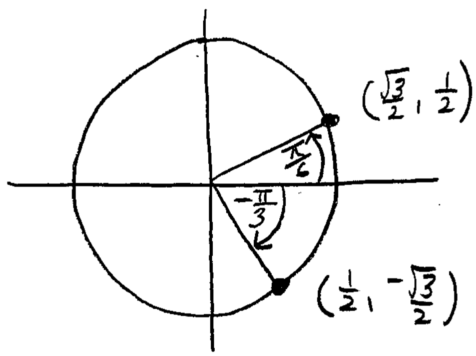
(b)[3 points] If θ is in the first quadrant and $\cos\left(\theta - \frac{\pi}{2}\right) = \frac{1}{2}$, determine $\sin\theta$.

$\theta - \frac{\pi}{2}$ must be in quadrant IV;

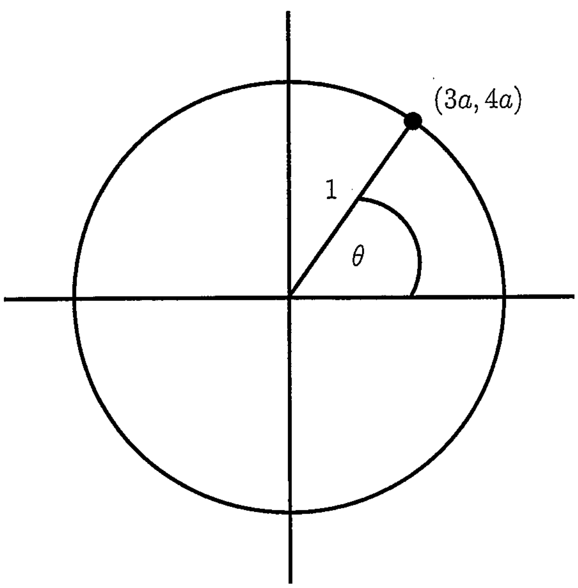
Since $\cos\left(\theta - \frac{\pi}{2}\right) = \frac{1}{2}$, $\theta - \frac{\pi}{2} = -\frac{\pi}{3}$

$\therefore \theta = -\frac{\pi}{3} + \frac{\pi}{2} = \frac{-2\pi + 3\pi}{6} = \frac{\pi}{6}$

$\therefore \sin\theta = \boxed{\frac{1}{2}}$



(c)[4 points] Use the information in the unit circle below to determine $\cos\theta$:



$(3a)^2 + (4a)^2 = 1$

$9a^2 + 16a^2 = 1$

$25a^2 = 1$

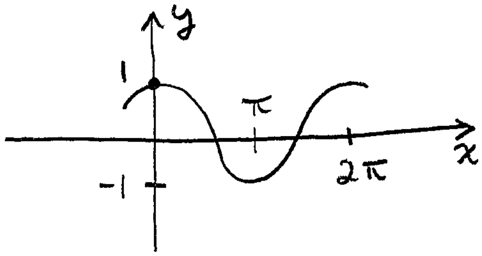
$a = \sqrt{\frac{1}{25}} = \frac{1}{5}$

$\therefore \cos\theta = 3a = \boxed{\frac{3}{5}}$

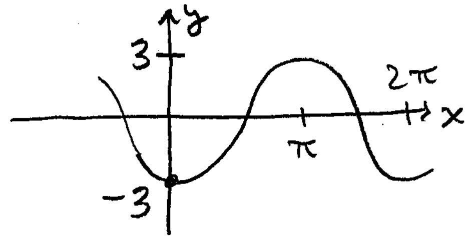
Question 5:

(a) **[7 points]** Carefully sketch the graph of $y = -3\cos\left(4x + \frac{\pi}{2}\right) + 1 = -3\cos\left[4\left(x + \frac{\pi}{8}\right)\right] + 1$

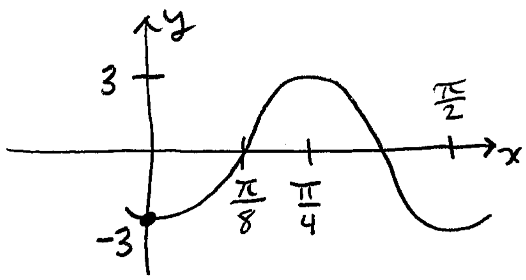
① $y = \cos(x)$



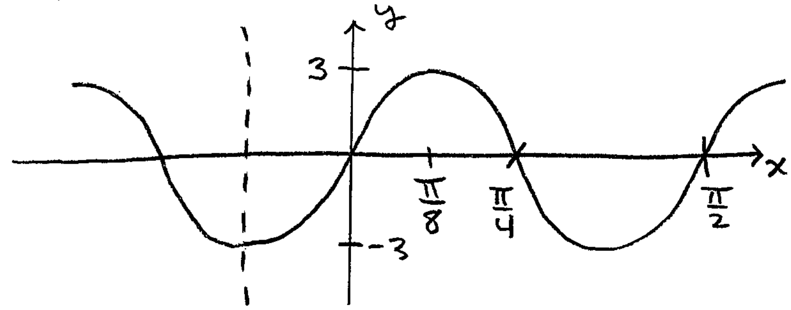
② $y = -3\cos(x)$



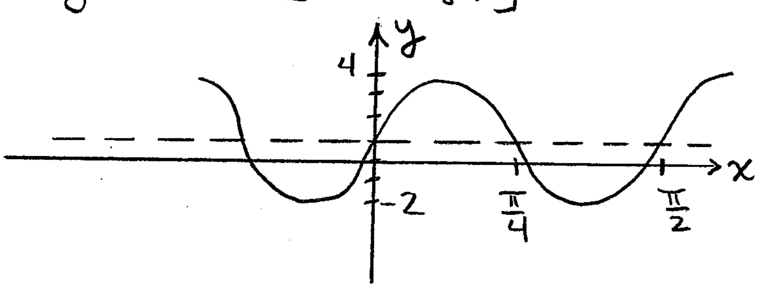
③ $y = -3\cos(4x)$



④ $y = -3\cos\left[4\left(x + \frac{\pi}{8}\right)\right]$



⑤ $y = -3\cos\left[4\left(x + \frac{\pi}{8}\right)\right] + 1$



(b) **[3 points]** State the period, amplitude and phase-shift of the function in (a).

Period $= \frac{2\pi}{4} = \frac{\pi}{2}$

Amplitude $= |-3| = 3$

Phase-Shift $= -\frac{\pi}{8}$.