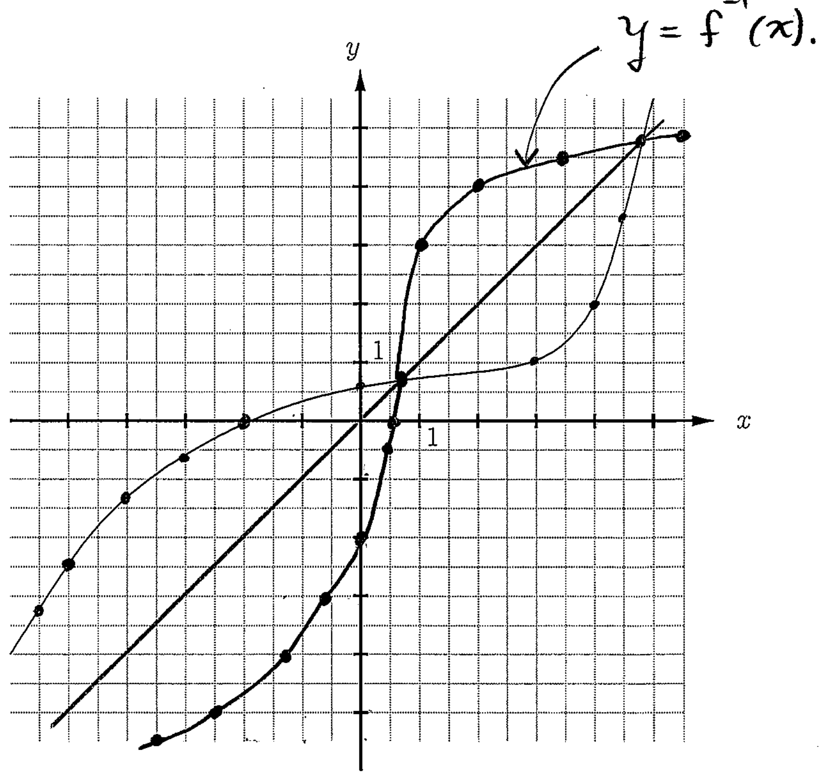
Question 1:

(a)[**5 points**] Let $f(x) = 1 - 3x^2$ and $g(x) = \sqrt{4-x}$. Find $(f \circ g)(x)$ and state the domain.

$$(f\circ g)(x) = f(g(x))$$
$$= 1 - 3(g(x))^2$$
$$= 1 - 3(\sqrt{4-x})^2 \quad \Big\} \ *$$
$$= 1 - 3(4-x)$$
$$= 3x - 11$$

Using $*$, must have $4 - x \geq 0$, so $x \leq 4$,
$\therefore$ domain is $(-\infty, 4]$.

(b)[**5 points**] Given below is the graph of $y = f(x)$. On the same set of axes carefully sketch the graph of $y = f^{-1}(x)$.



Question 2:

(a) [7 points] Let $f(x) = \dfrac{3x-5}{7x+2}$. Note that f is one-to-one. Find a formula for the inverse $f^{-1}(x)$.

$$y = \frac{3x-5}{7x+2}$$

$\underline{x \leftrightarrow y}$: $x = \dfrac{3y-5}{7y+2}$

$$7xy + 2x = 3y - 5$$

$$7xy - 3y = -2x - 5$$

$$y(7x-3) = -2x-5$$

$$y = \frac{-2x-5}{7x-3}$$

$$\therefore f^{-1}(x) = \frac{-2x-5}{7x-3}$$

For domain of f^{-1}, must have $7x-3 \neq 0$,

$\therefore x \neq \frac{3}{7}$

$\therefore$ domain of f^{-1} is $\{x \mid x \neq \frac{3}{7}\}$.

(b) [3 points] Use your result in part (a) to determine the domain and range of f.

For domain of $f(x) = \dfrac{3x-5}{7x+2}$, must have $7x+2 \neq 0$, i.e. $x \neq -\frac{2}{7}$.

$\therefore$ Domain of f is $\{x \mid x \neq -\frac{2}{7}\}$.

The range of f is the domain of f^{-1} from above: $\{y \mid y \neq \frac{3}{7}\}$.

Question 3:

(a)[7 points] Solve $e^{x^2} = e^{3x} \cdot \frac{1}{e^2}$.

$$e^{x^2} = e^{3x-2}$$

$$x^2 = 3x-2$$

$$x^2-3x+2=0$$

$$(x-2)(x-1) = 0$$

$$x=2, \quad x=1.$$

(b)[3 points] Calculate $\log_{\sqrt{5}}(2\pi)$ and round your final answer to three decimal places.

$$\log_{\sqrt{5}}(2\pi) = \frac{\ln(2\pi)}{\ln(\sqrt{5})} \approx 2.284$$

Question 4:

(a)[7 points] Solve $\log_2(x+3) = 3 - \log_2(x-4)$.

$$\log_2(x+3) + \log_2(x-4) = 3$$
$$\log_2[(x+3)(x-4)] = 3$$
$$(x+3)(x-4) = 2^3$$
$$x^2 - x - 12 = 8$$
$$x^2 - x - 20 = 0$$
$$(x-5)(x+4) = 0$$
$$x = 5,\ x = -4$$

Check:

$x = 5$:

$$\log_2(5+3) \quad 3 - \log_2(5-4)$$
$$\log_2 8 \quad 3 - \cancel{\log_2(1)}^{\,0}$$
$$3 = 3$$

$x = -4$:

$$\log_2(-4+3) \quad 3 - \log_2(-4-4)$$
$$\underbrace{\log_2(-1)} \quad 3 - \log_2(-8)$$

NOT DEFINED, so $x = -4$ is NOT A SOLUTION.

$\therefore$ $x = 5$ is the only solution.

(b)[3 points] Write as a single simplified logarithm: $\frac{1}{2}\ln(x+7) - \ln(4x^3) + 5\ln(2x)$.

$$\frac{1}{2}\ln(x+7) - \ln(4x^3) + 5\ln(2x)$$
$$= \ln(x+7)^{\frac{1}{2}} - \ln(4x^3) + \ln(2x)^5$$
$$= \ln(\sqrt{x+7}) - \ln(4x^3) + \ln(32x^5)$$
$$= \ln\left(\frac{\sqrt{x+7}}{4x^3}\right) + \ln(32x^5)$$
$$= \ln\left(\frac{\sqrt{x+7}\cdot 32x^5}{4x^3}\right) = \ln\left(8x^2\sqrt{x+7}\right).$$

Question 5: An isolated population of a particular insect grows according to the population growth function

$$P(t) = P_0 e^{kt} ,$$

where $P(t)$ is the population at time t, P_0 is the initial population, k is the population growth rate, and t is time in days.

(a) **[3 points]** If the population doubles in 10 days, what is the value of k? (Round to 4 decimals.)

Solve $P(10) = 2P_0$:

$$\cancel{P_0} e^{10k} = 2\cancel{P_0}$$

$$10k = \ln 2$$

$$k = \frac{\ln 2}{10} \approx 0.0693$$

(b) **[4 points]** If the initial population is 500 individuals, how long does it take the population to reach a size of 5100? (Round to the nearest day.)

Solve $P(t) = 5100$ where $P_0 = 500$:

$$500 e^{kt} = 5100$$

$$e^{kt} = \frac{51\cancel{00}}{5\cancel{00}}$$

$$kt = \ln\left(\frac{51}{5}\right)$$

$$\therefore t = \frac{\ln\left(\frac{51}{5}\right)}{k}$$

$$= \frac{\ln\left(\frac{51}{5}\right)}{\left[\frac{\ln 2}{10}\right]} \quad \text{from (a)}$$

$$\approx 34 \text{ days.}$$

(c) **[3 points]** Again, if the initial population is 500 individuals, how many days does it take for the population to increase from 700 to 1900 indiviuals? (Round to the nearest day.)

<u>Method 1:</u>

$$500 e^{kt_2} = 1900$$

$$500 e^{kt_1} = 700$$

$$\therefore \frac{5\cancel{00} e^{kt_2}}{5\cancel{00} e^{kt_1}} = \frac{1900}{700}$$

$$e^{k(t_2 - t_1)} = \frac{19}{7}$$

$$\therefore t_2 - t_1 = \frac{\ln\left(\frac{19}{7}\right)}{k} \approx \boxed{14 \text{ days}}$$

<u>Method 2:</u>

$$700 e^{kt} = 1900$$

$$e^{kt} = \frac{1900}{700} = \frac{19}{7}$$

$$t = \frac{\ln\left(\frac{19}{7}\right)}{k}$$

$$\approx 14 \text{ days.}$$