Math 152 S09N01 - Quiz 5 Name: SOLUTIONS
Mar 11 2009 Stu#:

(1) [5 points] Express $\tan\left(\sin^{-1} u\right)$ as an algebraic expression in u. Your final answer should not contain any trigonometric functions.

Let $\theta = \sin^{-1} u$.

$\therefore -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ and $\sin\theta = u$

$\sin^2\theta + \cos^2\theta = 1$, so $\cos\theta = \sqrt{1-\sin^2\theta} = \underbrace{\sqrt{1-u^2}}_{\text{positive square root since } \cos\theta \ge 0 \text{ for } -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}}$

$$\therefore \tan(\sin^{-1} u) = \tan\theta = \frac{\sin\theta}{\cos\theta} = \boxed{\frac{u}{\sqrt{1-u^2}}}$$

(2) [5 points] Find the exact value of $\sec\left(-\frac{\pi}{12}\right)$.

$$\sec\left(-\frac{\pi}{12}\right) = \frac{1}{\cos\left(-\frac{\pi}{12}\right)}$$

$$= \frac{1}{\cos\left(\frac{3\pi}{12} - \frac{4\pi}{12}\right)}$$

$$= \frac{1}{\cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right)}$$

$$= \frac{1}{\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)}$$

$$= \frac{1}{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)}$$

$$= \boxed{\frac{2\sqrt{2}}{1+\sqrt{3}}} \text{ or } \frac{2\sqrt{2}(1-\sqrt{3})}{-2} = \boxed{\sqrt{2}(\sqrt{3}-1)}$$

(3) [**5 points**] Solve for $0 \le \theta < 2\pi$:

$$3(1 - \cos\theta) = \sin^2\theta$$

$$3 - 3\cos\theta = 1 - \cos^2\theta$$

$$\cos^2\theta - 3\cos\theta + 2 = 0$$

$$(\cos\theta - 2)(\cos\theta - 1) = 0$$

$\therefore \cos\theta - 2 = 0$, $\cos\theta - 1 = 0$

$\underbrace{\cos\theta = 2}_{\text{no solutions.}}$, $\cos\theta = 1$

$\boxed{\theta = 0}$