

(1) [5 points] Let $f(x) = \frac{3}{x-1}$ and $g(x) = \frac{2}{x}$. Find $(f \circ g)(x)$ and state the domain.

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) = \frac{3}{\left(\frac{2}{x}\right) - 1} \quad \} * \\ &= \frac{3x}{2-x} \end{aligned}$$

Using *, $x \neq 0$, $\frac{2}{x} - 1 \neq 0$

$$\therefore x \neq 0, x \neq 2$$

\therefore domain is $\{x \mid x \neq 0, x \neq 2\}$.

(2) [5 points] Let $H(x) = \sqrt{x^2 + 1}$. Find functions f and g so that $H = f \circ g$.

$$\text{Let } g(x) = x^2 + 1$$

$$\therefore f(x) = \sqrt{x}$$

Another solution: Let $g(x) = x^2$
 $\therefore f(x) = \sqrt{x+1}$

(3) [5 points] The function $f(x) = \frac{2x}{3x-1}$ is one-to-one. Find f^{-1} .

$$y = \frac{2x}{3x-1}$$

$$x \leftrightarrow y : x = \frac{2y}{3y-1}$$

$$(3y-1)x = 2y$$

$$3xy - x = 2y$$

$$3xy - 2y = x$$

$$y(3x-2) = x$$

$$y = \frac{x}{3x-2}$$

$$\therefore f^{-1}(x) = \frac{x}{3x-2} .$$