Question 1 [10 points]:

(a) [5] The one to one function $f(x) = \sqrt{\dfrac{2x-1}{5}} + 3$ has domain $[1/2, \infty)$ and range $[3, \infty)$. Determine $f^{-1}(x)$ and state its domain and range.

(b) [3] Express as a single logarithm: $\log_2 x + 3\log_2(x^2+1) - \log_2 5$

(c) [2] Calculate $\log_{\sqrt{2}} 7$ (round your answer to two decimal places).

Question 2 [10 points]: Solve the following equations for x:

(a) [5] $2^{3x^2} = 4 \cdot 2^{5x}$

(b) [5] $\ln(x) = \ln(35) - \ln(x-2)$

Question 3 [10 points]:

(a) [5] The half-life of iodine-131 is eight days. A sample originally containing 10 g of iodine-131 now contains only 0.3 g. How old is it? Round your answer to two decimal places. (Recall that the amount A of radioactive material present at time t is given by $A(t) = A_0 e^{kt}$ where A_0 is the original amount of radioactive material and k is a negative number.)

(b) [5] The population of one country grows according to the model $P_1(t) = 12{,}000e^{0.02t}$, while that of a second country grows according to the model $P_2(t) = 4000e^{0.025t}$. Here t represents time in years. If $t = 0$ corresponds to the present, in how many years time will the two populations be equal?

Question 4 [10 points]:

(a) [1] Convert -330° to radians.

(b) [2] Determine the exact value of $\cos\left(\dfrac{49\pi}{6}\right)$.

(c) [3] Determine the exact value of $\cos\left(\dfrac{7\pi}{4}\right)\csc\left(\dfrac{7\pi}{3}\right)$.

(d) [4] Suppose $\sin\theta = 2/5$ where $\dfrac{\pi}{2} < \theta < \pi$. Determine $\tan\theta$.

Question 5 [10 points]:

(a) [7] Carefully sketch the graph of $y = 2\sin(3x - \pi) - 2$.

(b) [3] State the period, amplitude and phase-shift of the function sketched in part (a).

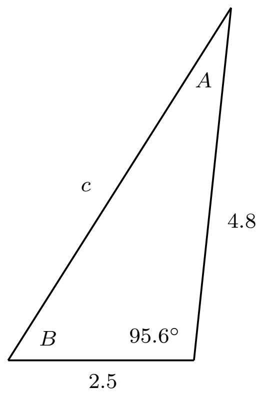
Question 6 [10 points]: Find all solutions in the interval $[0, 2\pi)$:

(a) [5] $\sin(2\theta)\sin(\theta) = \cos(\theta)$

(b) [5] $4(1+\sin(\theta)) = \cos^2(\theta)$

Question 7 [10 points]:

(a) [5] Solve for all missing sides and angles in the following triangle. Round final answers to one decimal place.



(b) [5] A triangle has angle $A = 30°$, side $c = 5$ and side $a = 3$. Determine the two possible values for angle C.

Question 8 [10 points]:

(a) [**5**] An observer on the ground is looking up at a 2 m tall statue mounted atop a building. The angle of elevation to the bottom of the statue is $50°$, while that to the top of the statue is $53°$. How tall is the building? Round your final answer to one decimal place.

(b) [**5**] Two people leave from the same point: one walks north at 5 km/h, the other walks at 4 km/h at a bearing of $S30°E$. What is the distance between the two people three hours later? Again, round your final answer to one decimal place.

Question 9 [10 points]: For this question use

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$\mathbf{D} = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 0 & 5 \end{bmatrix}, \mathbf{E} = \begin{bmatrix} 2 & 1 \\ 8 & -1 \\ 6 & 5 \end{bmatrix}, \mathbf{F} = \begin{bmatrix} 5 & -1 \\ -4 & 0 \\ 2 & 3 \end{bmatrix}.$$

(a) [**2**] Compute $2A - 5C$.

(b) [**4**] Compute $D(F - E)$.

(c) [**4**] Compute A^{-1} .

Question 10 [10 points]: Solve the following system of equations **using matrix reduction** (no credit will be given for using any other method):

$$\begin{aligned} x + y + z &= 1 \\ -2x + y + z &= -2 \\ 3x + 6y + 6z &= 5 \end{aligned}$$

Question 11 [10 points]:

(a) [**3**] Find a_{20} in the arithmetic sequence $5, 9/2, 4, 7/2, \ldots$.

(b) [**3**] If $a_5 = 2/3$ and $a_7 = 10/3$ in a geometric sequence, what must be the common ratio r if it is a positive number?

(c) [**4**] The arithmetic series $2 + \cdots + 18 = 500$. How many terms are in the series?

Question 12 [10 points]: A person would like to accumulate \$1,000,000 by his 60^{th} birthday so he can retire early. The plan is to make equal deposits into a fund on each birthday beginning with the 20^{th} and ending with the final payment made on the 60^{th}. If all deposits are invested in a fund paying 5% interest compounded annually, how large must the equal annual deposits be in order to achieve the \$1,000,000 goal? Recall that P dollars invested at $i\%$ interest compounded annually accumulates to an amount $A = P\left(1 + \frac{i}{100}\right)^n$ by the end of n years.