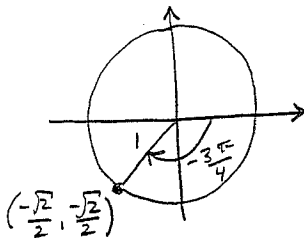
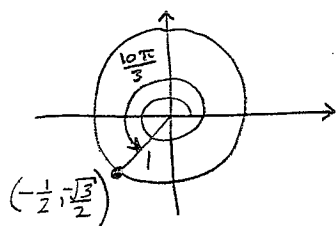


Question 1:

(a)[2 points] Convert -405° to radians.

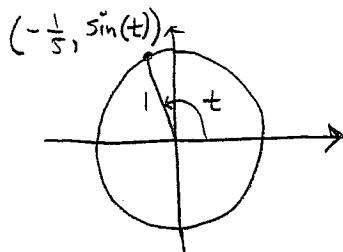
$$(-405^\circ) \left(\frac{\pi \text{ radians}}{180^\circ} \right) = \boxed{\frac{-9\pi}{4}}$$

(b)[4 points] Find the exact value of $\sec(10\pi/3) \sin(-3\pi/4)$.



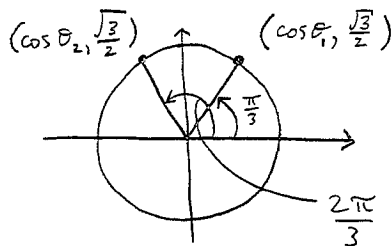
$$\begin{aligned} \therefore \sec\left(\frac{10\pi}{3}\right) \sin\left(-\frac{3\pi}{4}\right) &= \frac{1}{\cos\left(\frac{10\pi}{3}\right)} \cdot \sin\left(-\frac{3\pi}{4}\right) \\ &= \frac{1}{(-\frac{1}{2})} \cdot \frac{-\sqrt{2}}{2} \\ &= (-2) \left(\frac{-\sqrt{2}}{2} \right) \\ &= \boxed{\sqrt{2}} \end{aligned}$$

(c)[4 points] If $\cos(t) = -1/5$, find all possible values of $\sin(t)$.



$$\begin{aligned} \therefore \left(-\frac{1}{5}\right)^2 + \sin^2(t) &= 1 \\ \sin(t) &= \pm \sqrt{1 - \left(-\frac{1}{5}\right)^2} \\ &= \pm \sqrt{1 - \frac{1}{25}} \\ &= \pm \sqrt{\frac{24}{25}} \\ &= \boxed{\pm \frac{2\sqrt{6}}{5}} \end{aligned}$$

Question 2:

(a) [3 points] Find all angles $0 \leq \theta < 2\pi$ such that $\sin \theta = \sqrt{3}/2$.

$$\therefore \theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

(b) [3 points] Find the exact value of $\sin(11\pi/12)$ (note: $2/3 + 1/4 = 11/12$).

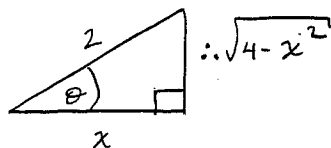
$$\begin{aligned} \sin\left(\frac{11\pi}{12}\right) &= \sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) \\ &= \sin\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) \\ &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{2}(\sqrt{3}-1)}{4} \end{aligned}$$

(c) [4 points] Simplify to an expression which does not contain trigonometric functions:

$$\sin(\arccos(x/2))$$

$$\text{let } \theta = \arccos\left(\frac{x}{2}\right),$$

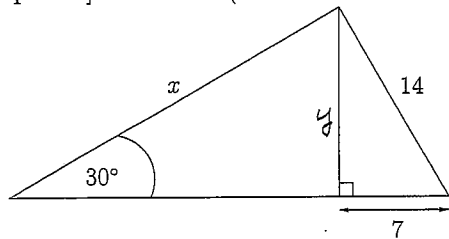
$$\therefore \cos \theta = \frac{x}{2}, \text{ where } 0 \leq \theta \leq \pi$$



$$\therefore \sin(\arccos(x/2)) = \sin(\theta) = \frac{\sqrt{4-x^2}}{2}$$

Question 3:

(a) [4 points] Solve for x (round final answer to one decimal):

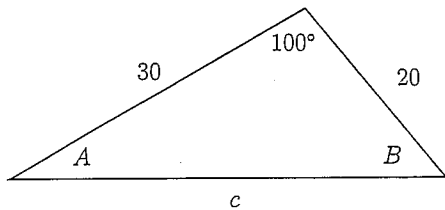


$$y = \sqrt{14^2 - 7^2} = \sqrt{147} = 7\sqrt{3}$$

$$\sin(30^\circ) = \frac{y}{x} \quad \therefore x = \frac{y}{\sin(30^\circ)} = \frac{7\sqrt{3}}{\frac{1}{2}}$$

$$= 14\sqrt{3} \\ \doteq 24.2$$

(b) [4 points] Find all remaining sides and angles in the following figure (round final answers to one decimal):



$$c^2 = a^2 + b^2 - 2ab \cos C \\ = (20)^2 + (30)^2 - 2(20)(30) \cos(100)$$

$$c = \sqrt{1300 - 1200 \cos(100^\circ)} \doteq 38.8$$

$$\frac{\sin A}{20} = \frac{\sin(100^\circ)}{c} \quad \therefore A = \sin^{-1} \left[\frac{20 \sin(100^\circ)}{\sqrt{1300 - 1200 \cos(100^\circ)}} \right]$$

$$A \doteq 30.5^\circ$$

$$\frac{\sin B}{30} = \frac{\sin(100^\circ)}{c} \quad \therefore B = \sin^{-1} \left[\frac{30 \sin(100^\circ)}{\sqrt{1300 - 1200 \cos(100^\circ)}} \right] \doteq 49.5^\circ$$

(c) [2 points] Find the exact value of $\log_{\frac{1}{2}} 16$

$$\log_{\frac{1}{2}} 16 = \frac{\log_2 16}{\log_2 \frac{1}{2}} = \frac{4}{-1} = -4$$

Question 4:

(a)[2 points] Find the x intercept of the graph of $y = \log_7(2x-3) - 2$.

$$0 = \log_7(2x-3) - 2$$

$$2 = \log_7(2x-3)$$

$$49 = 2x-3$$

$$x = \frac{49+3}{2} = 26, \quad \boxed{\therefore (26, 0)}$$

(b)[2 points] Simplify:

$$\ln\left(\frac{1}{4}e^{3x}\right) - \ln(e^{2x}) + \ln 4$$

$$= \ln\left(\frac{1}{4}\right) + \ln(e^{3x}) - \ln(e^{2x}) + \ln 4$$

$$= \cancel{\ln 1} - \cancel{\ln 4} + 3x - 2x + \cancel{\ln 4}$$

$$= \boxed{x}$$

(c)[3 points] Let $A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -5 \\ -9 & 2 \end{bmatrix}$, and $C = \begin{bmatrix} -2 & 2 \\ 4 & -1 \end{bmatrix}$. Compute $(B - 2C)A^T$.

$$B - 2C = \begin{bmatrix} 7 & -9 \\ -17 & 4 \end{bmatrix}$$

$$\therefore (B - 2C)A^T = \begin{bmatrix} 7 & -9 \\ -17 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix} = \boxed{\begin{bmatrix} 23 & -18 \\ -38 & 8 \end{bmatrix}}$$

(d)[3 points] Let $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$. Find A^{-1} .

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{array} \right]$$

$$\therefore A^{-1} = \boxed{\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}}$$

$$(-1)R_2: \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{array} \right]$$

$$(-2)R_2 + R_1: \left[\begin{array}{cc|cc} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -1 \end{array} \right]$$

Question 5:

(a)[3 points] Find the 11th term of the arithmetic sequence $\frac{7}{6}, \frac{5}{6}, \dots$

$$a_n = a + (n-1)d$$

$$a = \frac{7}{6},$$

$$d = \frac{5}{6} - \frac{7}{6} = \frac{-2}{6} = \frac{-1}{3}$$

$$n = 11$$

$$\therefore a_{11} = \frac{7}{6} + 10\left(\frac{-1}{3}\right)$$

$$= \frac{7}{6} - \frac{20}{6}$$

$$= \boxed{\frac{-13}{6}}$$

(b)[3 points] A geometric sequence has $a_3 = 1/2$ and $a_8 = -512$. What is a_6 ?

$$\left. \begin{array}{l} a_3 = \frac{1}{2} = ar^2 \\ a_8 = -512 = ar^7 \end{array} \right\} \therefore \frac{ar^7}{ar^2} = \frac{-512}{\left(\frac{1}{2}\right)}$$

$$r^5 = -1024$$

$$\therefore r = -4$$

$$\therefore a_6 = a_3 \cdot r^3$$

$$= \left(\frac{1}{2}\right)(-4)^3$$

$$= \frac{-64}{2} = \boxed{-32}$$

(c)[4 points] An arithmetic series has first term 7, last term -47 and common difference between terms of $d = -3$. Find the sum of the series.

$$a_1 = 7$$

$$d = -3$$

$$a_n = a + (n-1)d = -47$$

$$7 + (n-1)(-3) = -47$$

$$n-1 = \frac{-47-7}{-3}$$

$$\therefore n = 19$$

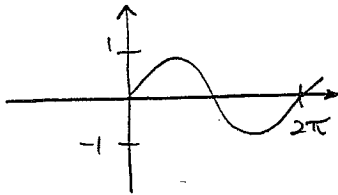
$$\therefore S_{19} = 19 \left(\frac{7 + (-47)}{2} \right) = \boxed{-380}$$

Question 6:

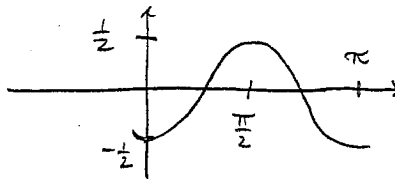
(a) [7 points] Carefully sketch the graph of $f(x) = \frac{1}{2} \sin\left(2x - \frac{\pi}{4}\right) - \frac{1}{2}$ showing at least one complete cycle of the function. Label and indicate the scale on your axes.

$$y = \frac{1}{2} \sin\left[2\left(x - \frac{\pi}{4}\right)\right] - \frac{1}{2}$$

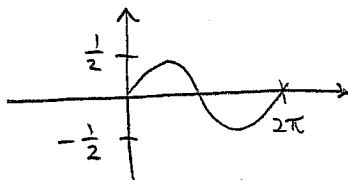
① $y = \sin(x)$:



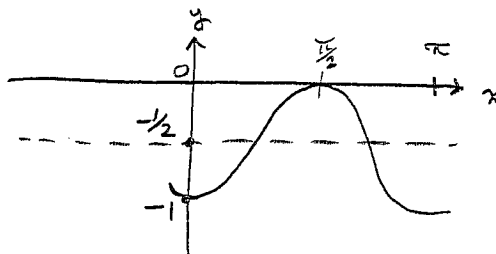
④ $y = \frac{1}{2} \sin\left[2\left(x - \frac{\pi}{4}\right)\right]$



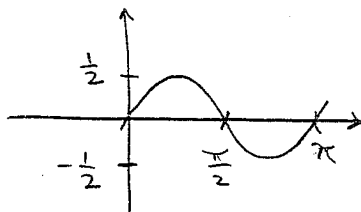
② $y = \frac{1}{2} \sin(x)$:



⑤ $y = \frac{1}{2} \sin\left[2\left(x - \frac{\pi}{4}\right)\right] - \frac{1}{2}$



③ $y = \frac{1}{2} \sin(2x)$



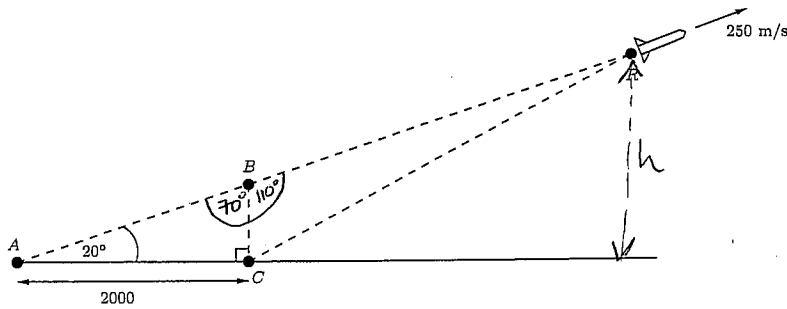
(b) [3 points] State the amplitude, period and phase-shift of the function graphed in (a).

Amplitude = $\left|\frac{1}{2}\right| = \boxed{\frac{1}{2}}$

Period = $\frac{2\pi}{2} = \boxed{\pi}$

Phase-shift = $\boxed{\frac{\pi}{4}}$

Question 7: A rocket traveling at 250 metres per second is climbing at an angle of 20° as shown in the figure below. A radar station at point C located 2000 metres from the launch point A is tracking the rocket.



(a)[3 points] What is the distance from the launch point A to the rocket at R three seconds after the rocket passes through point B ? (round your answer to the nearest metre.)

$$BR = \left(250 \frac{\text{m}}{\text{s}}\right)(3\text{s}) = 750\text{ m}$$

$$AB = \frac{2000}{\cos(20^\circ)}$$

$$\therefore AR = 750 + \frac{2000}{\cos(20^\circ)} \doteq \boxed{2878\text{ m}}$$

(b)[4 points] How far is the rocket at R from the radar station C at this same instant? (round your answer to the nearest metre.)

$$BC = 2000 \tan(20^\circ)$$

$$(CR)^2 = (BC)^2 + (BR)^2 - 2(BC)(BR)\cos(110^\circ)$$

$$\therefore CR = \sqrt{(2000 \tan(20^\circ))^2 + (750)^2 - 2(2000 \tan(20^\circ))(750)\cos(110^\circ)}$$

$$\doteq \boxed{1211\text{ m}}$$

(c)[3 points] How high above the ground is the rocket at this same instant? (round your answer to the nearest metre.)

$$h = AR \sin(20^\circ)$$

$$= \left[750 + \frac{2000}{\cos(20^\circ)}\right] \sin(20^\circ)$$

$$\doteq \boxed{984\text{ m}}$$

Question 8: One population has size $P_1(t)$ at time t years given by $P_1(t) = 1000e^{0.05t}$. A second population has size $P_2(t)$ at time t years given by $P_2(t) = 800e^{0.08t}$.

(a)[3 points] What is the doubling time of the first population? (round your answer to one decimal.)

$$\begin{aligned}2000 &= 1000 e^{0.05t} \\2 &= e^{0.05t} \\ \ln 2 &= 0.05t \\ t &= \frac{\ln 2}{0.05} \doteq \boxed{13.9 \text{ years}}\end{aligned}$$

(b)[3 points] How many years does it take the second population to reach 2500 in size? (round your answer to one decimal.)

$$\begin{aligned}2500 &= 800 e^{0.08t} \\ \frac{2500}{800} &= e^{0.08t} \\ 0.08t &= \ln\left(\frac{25}{8}\right) \\ \therefore t &= \frac{\ln\left(\frac{25}{8}\right)}{0.08} \doteq \boxed{14.2 \text{ years}}\end{aligned}$$

(c)[4 points] At what time t will both populations be equal in size? (round your answer to one decimal.)

$$\begin{aligned}1000 e^{0.05t} &= 800 e^{0.08t} \\ \frac{1000}{800} &= \frac{e^{0.08t}}{e^{0.05t}} \\ \frac{5}{4} &= e^{0.03t} \\ \ln\left(\frac{5}{4}\right) &= 0.03t \\ \therefore t &= \frac{\ln\left(\frac{5}{4}\right)}{0.03} \doteq \boxed{7.4 \text{ yrs}}\end{aligned}$$

Question 9:

(a) [5 points] Solve for x :

$$\log_{10}(3x) - \log_{10}(x+1) = \log_{10} x.$$

$$\log_{10}(3x) = \log_{10}(x) + \log_{10}(x+1)$$

$$\log_{10}(3x) = \log_{10}[x(x+1)]$$

$$3x = x^2 + x$$

$$\therefore x^2 - 2x = 0.$$

$$x(x-2) = 0$$

$$x = 0, x = 2.$$

Check:

$x=0$: $\underbrace{\log_{10}(3 \cdot 0) - \log_{10}(0+1)}_{\text{not defined!}} \left\{ \log_{10}(0) \right\} \therefore x=0 \text{ is } \underline{\text{not}}$
a solution.

$x=2$: $\log_{10}(3 \cdot 2) - \log_{10}(2+1) \left\{ \begin{array}{l} \log_{10}(2) \\ \log_{10}(6) - \log_{10}(3) \\ \log_{10}(\frac{6}{3}) \\ \log_{10}(2) \end{array} \right\} \therefore x=2 \text{ is the}$
 $\log_{10}(2) \equiv \log_{10}(2)$ only solution.

(b) [5 points] Find all solutions $0 \leq t < 2\pi$ to

$$2\sin^2(t) + \sin(t) - 1 = 0.$$

Let $w = \sin(t)$: $2w^2 + w - 1 = 0$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-1 \pm 3}{4}$$

$$= \frac{1}{2}, -1$$

$\therefore \sin(t) = \frac{1}{2}$, $\sin t = -1$

$\therefore t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

Question 10 [10 points]: Solve the following system of equations using matrix reduction (no credit will be given for using any other method):

$$\begin{aligned}5x - 10y + 5z &= -15 \\ -5x + 8y - 7z &= -5 \\ 10x - 18y + 13z &= -3\end{aligned}$$

$$\left[\begin{array}{ccc|c} 5 & -10 & 5 & -15 \\ -5 & 8 & -7 & -5 \\ 10 & -18 & 13 & -3 \end{array} \right]$$

$$\frac{1}{5}R_1: \left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ -5 & 8 & -7 & -5 \\ 10 & -18 & 13 & -3 \end{array} \right]$$

$5R_2 + R_1:$

$-10R_1 + R_3:$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 0 & -2 & -2 & -20 \\ 0 & 2 & 3 & 27 \end{array} \right]$$

$-\frac{1}{2}R_2:$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 0 & 1 & 1 & 10 \\ 0 & 2 & 3 & 27 \end{array} \right]$$

$2R_2 + R_1:$

$-2R_2 + R_3:$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 17 \\ 0 & 1 & 1 & 10 \\ 0 & 0 & 1 & 7 \end{array} \right]$$

$-3R_3 + R_1:$

$-R_3 + R_2:$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 7 \end{array} \right]$$

$$\begin{aligned} \therefore x &= -4 \\ y &= 3 \\ z &= 7 \end{aligned}$$