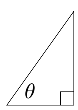
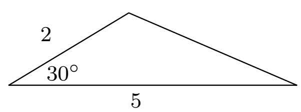
Math 152
Practice Problems for Final Exam

The following problems are good practice for the final exam. Some of these problems are challenging but within the scope of the material we have covered. The problems on the final exam may of course be completely different.

Shorter Problems

1. Sketch the graph of $f(x) = 5 - 2^x$.
2. Solve for x: $e^{x^2-3} = e^{2x}$.
3. Sketch the graph of $f(x) = -\ln(x) + 2$.
4. Evaluate $\log_5 75 - \log_5 3$.
5. Express as a single logarithm: $\ln x - 4(\ln(x+2) + \ln(x-2))$.
6. Convert π/a radians to degrees.
7. Find $\cos(95\pi/6 - 10\pi)\tan(-50\pi/3)$ exactly.
8. If $\sin\alpha = 2/\sqrt{7}$ and $\pi/2 \le \alpha \le \pi$, find $\tan\alpha$.
9. If $\cos\alpha = a/b$, what is $\csc^2\alpha$?
10. If $\sec\theta = \sqrt{5}$ in 

find all remaining sides and angles. (Note figure not to scale).
11. Find all remaining sides and angles of 

(Note figure not to scale).
12. Triangle ABC has angle $B = 25°$, the side opposite B is $b = 7$, and the angle opposite angle C is $c = 13$. Find all possible values for the remaining sides and angles.
13. Given that $\sin(\pi/10) = (\sqrt{5} - 1)/4$, find $\sin(\pi/5)$ exactly.
14. If $\sin\theta = x/4$ where $0 \le \theta \le \pi/2$, find an expression for $\sin(2\theta)$ which does not involve trigonometric functions.
15. Compute $\tan[\sin^{-1}(-5/7)]$.
16. Simplify $\sin^{-1}[\sin(-81\pi/10)]$.
17. Suppose $\mathbf{A}$ and $\mathbf{B}$ are column matrices with 5 entries each. What is the size of $\mathbf{A}\mathbf{I_1}(\mathbf{A} + \mathbf{B})^{\mathrm{T}}\mathbf{I_5}\mathbf{B}$?
18. Suppose $\mathbf{A}$ and $\mathbf{B}$ are $n \times n$ matrices. Simplify

$$\left(\frac{1}{2}\mathbf{A} + \frac{1}{2}\mathbf{B}\right)(2\mathbf{A} - 2\mathbf{B}) - \mathbf{BA}$$

19. Find the sum of the first 100 terms of the sequence whose terms are given by $a_n = e^{-n}$, $n = 0, 1, 2, \ldots$

20. The sum of the first 12 terms of an arithmetic sequence is 156. What is the sum of the second through 11$^{\text{th}}$ terms?

Longer Problems

1. Solve for x: $\log(8x) - \log(1 + \sqrt{x}) = 2$

2. A culture of bacteria has size $N = 250e^{kt}$ after t hours. If the culture contains 430 bacteria after 10 hours, what is the doubling time of the population?

3. The size of a certain population at time t years is $P_1(t) = 2000e^{0.06t}$, while the size at time t of a second population is $P_2(t) = 2400e^{0.055t}$. In both cases $t = 0$ corresponds to the present. At what time t will both populations be equal in size?

4. Find all values of θ in $[0, \pi/2]$ for which

$$2\sin^2\theta - 3\sin\theta + 1 = 0\ .$$

   Hint: let $x = \sin\theta$.

5. Graph $y = -3\cos(4x - \pi) + 2$ and state the period, amplitude and phase shift.

6. Graph $y = \dfrac{1}{2}\sin(2\pi x + \pi) - 1$ and state the period, amplitude and phase shift.

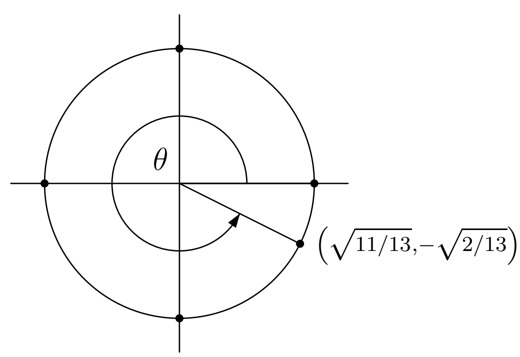
7. Consider the angle θ in the unit circle below:

   (a) Find $\cos\theta$.

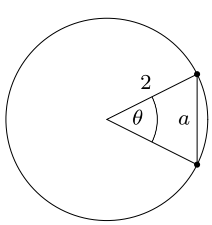
   (b) Find $\csc\theta$.

   (c) Find $\sin(\theta + \pi)$.

   (d) Find the coordinates of the point on the circle corresponding to the angle $\theta + \pi/2$.

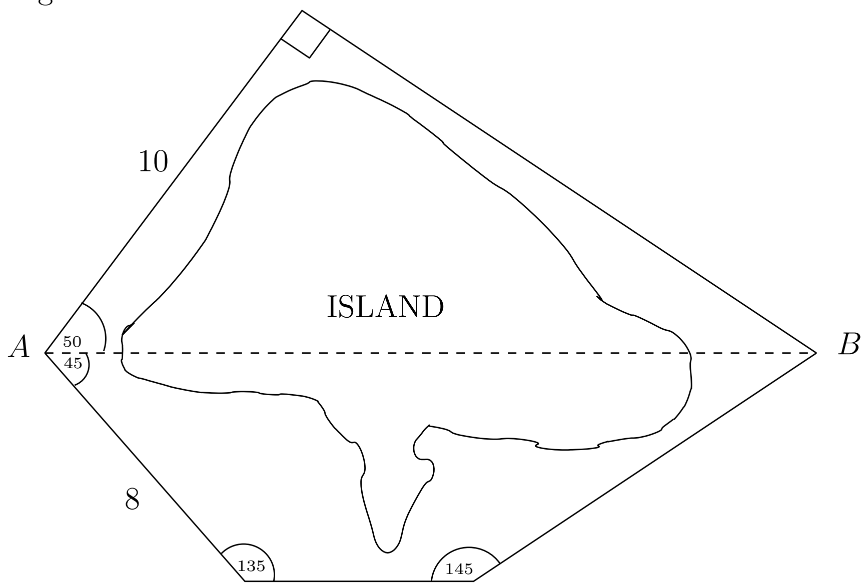


8. Find an expression for a in terms of θ using the figure below:

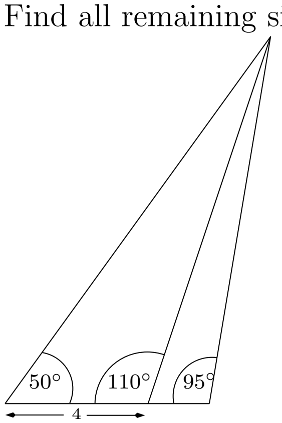


9. You measure the angle of elevation to the top of a mountain to be $40°$. You then walk 300 meters directly away from the mountain and take another measurement, this time finding the angle of elevation to be $30°$. How tall is the mountain?

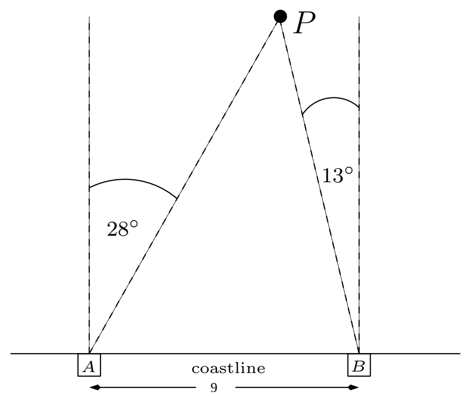
10. A ship at position A wishes to navigate around an island to point B using one of the two routes shown. Assuming the ship travels at the same constant speed over both routes, which should be chosen to complete the journey as quickly as possible? The angles shown are in degrees.



11. Find all remaining sides and angles in the following:



12. A plane makes an emergency landing in the ocean at point P. Rescue stations 9 kilometers apart at A and B both spot the plane and take the angle bearings shown. The rescue boat at A has a maximum speed of 40 knots, while that at B can do at most 35 knots. Which station should respond to the downed plane in order to reach the scene at quickly as possible?



13. Find the value of $\tan(\pi/8)$ exactly (trigonometric identities will help here.)

14. If $\sin a = 2/3$ and $\sin b = 1/7$, where both a and b are angles in $[0, \pi/2]$, compute and simplify $\cos(a-b)\cos(a+b)$.

15. Compute and simplify $\sin\left[\sin^{-1}(1/3) + \sin^{-1}(1/4)\right]$

16. Let
$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 2 \\ 1 & 1 & 3 \end{bmatrix}$$
   (a) Find $\mathbf{A^{-1}}$.
   (b) Use the inverse in part (a) to solve the system of equations $\mathbf{AX} = \mathbf{Y}$ where $\mathbf{X}^{\mathrm{T}} = \begin{bmatrix} x_{11} & x_{21} & x_{31} \end{bmatrix}$ and $y_{11} = 1$, $y_{21} = 4$, $y_{31} = 1$.

17. Let
$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -3 \\ 3 & -1 & -2 \\ 2 & 3 & -5 \end{bmatrix}$$
Suppose also that $\mathbf{B}$ is the matrix of size 3×1 whose entries are $b_{11} = -2$, $b_{21} = 1$ and $b_{31} = -3$.
   (a) Find the inverse of $\mathbf{A}$ or show that it does not exist.
   (b) Find all solutions to $\mathbf{AX} = \mathbf{B}$ where $\mathbf{X}^{\mathrm{T}} = \begin{bmatrix} x & y & z \end{bmatrix}$.

18. Let
$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 4 & -6 \\ -2 & 3 \end{bmatrix}.$$
   (a) Compute and simplify $(\mathbf{A} - 2\mathbf{I_2})(\mathbf{B} + 3\mathbf{I_2})$.
   (b) Find the inverse of the matrix from part (a) if possible.

19. The sum of three consecutive terms $x - d, x, x + d$ of an arithmetic sequence is 30, while their product is 360. Find the terms.

20. $\$P$ is deposited at the beginning of each year for 20 years into an investment paying 7% compounded annually. At the end of 20 years, the accumulated value is S_1. Deposits of $\$P$ are also made at the beginning of each year for 20 years into a second investment paying 8% compounded annually, and at the end of the 20 years the second investment has accumulated to a value of S_2. What is S_2/S_1?