Math 121 S09N03 - Quiz 5
Mar 13 2009

Name: SOLUTIONS
Stu#:

(1) [**5 points**] Differentiate:

$$y = 2^{\sin(\pi x)}$$

$$y' = 2^{\sin(\pi x)} \cdot \ln 2 \cdot \frac{d}{dx}\left[\sin(\pi x)\right]$$

$$= 2^{\sin(\pi x)} \ln 2 \cdot \cos(\pi x) \cdot \pi$$

(2) [**5 points**] Use logarithmic differentiation to find the derivative:

$$y = (\cos x)^x$$

$$\ln y = \ln\left[(\cos x)^x\right]$$

$$\ln y = x \ln\left[\cos x\right]$$

$$\therefore \frac{1}{y} y' = 1 \cdot \ln\left[\cos x\right] + x \frac{1}{\cos x} \cdot (-\sin x)$$

$$\therefore y' = (\cos x)^x \left[\ln(\cos x) - \frac{x \sin x}{\cos x}\right]$$

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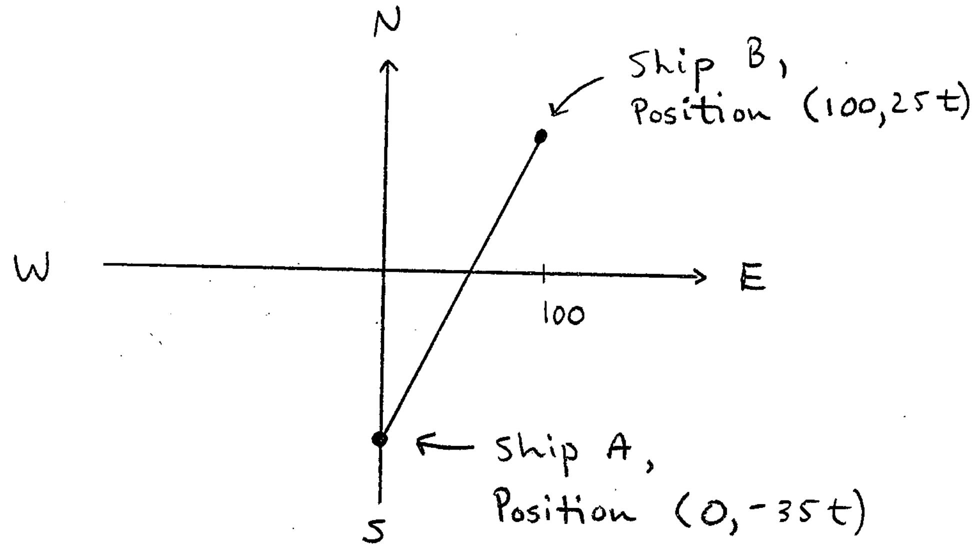
(3) [5 points] At 12:00 ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing four hours later?



$L(t)$ = distance between ships at time t hrs.

$$L(t) = \left[(100-0)^2 + (25t-(-35t))^2\right]^{\frac{1}{2}}$$

Find $\frac{dL}{dt}$ when $t=4$.

$$L(t) = \left[100^2 + 60^2 t^2\right]^{\frac{1}{2}}$$

$$\therefore \quad \frac{dL}{dt} = \frac{1}{2}\left[100^2 + 60^2 t^2\right]^{-\frac{1}{2}} \cdot 2(60^2)\, t$$

At $t=4$, $\frac{dL}{dt} = \frac{1}{\cancel{2}}\left[100^2 + 60^2(4)^2\right]^{-\frac{1}{2}} \cdot \cancel{2}(60^2) \cdot 4 = \frac{14400}{\left[100^2 + 240^2\right]^{\frac{1}{2}}}$

$\therefore$ At $t=4$ hrs distance is increasing at $\frac{14400}{\left[100^2+240^2\right]^{1/2}} \frac{km}{hr}$.