

(1) [5 points] Evaluate the limit, if it exists:

$$\lim_{x \rightarrow -4} \frac{\frac{1}{4} + \frac{1}{x}}{4+x} \sim \frac{0}{0}$$

$$\lim_{x \rightarrow -4} \frac{\left(\frac{1}{4} + \frac{1}{x}\right)}{4+x}$$

$$= \lim_{x \rightarrow -4} \frac{\left(\frac{4+x}{4x}\right)}{\frac{(4+x)}{1}}$$

$$= \lim_{x \rightarrow -4} \frac{\cancel{(4+x)}}{4x} \cdot \frac{1}{\cancel{(4+x)}}$$

$$= \frac{-1}{16}$$

(2) [5 points] Use the Squeeze Theorem to show that  $\lim_{x \rightarrow 0} x^4 \cos\left(\frac{2}{x}\right) = 0$ .

$$-x^4 \leq x^4 \cos\left(\frac{2}{x}\right) \leq x^4$$

Since  $\lim_{x \rightarrow 0} (-x^4) = 0 = \lim_{x \rightarrow 0} (x^4)$ , by the Squeeze Theorem,

$$\lim_{x \rightarrow 0} x^4 \cos\left(\frac{2}{x}\right) = 0.$$

(3) [5 points] Find the limit:  $\lim_{t \rightarrow 0} \frac{\tan(6t)}{\sin(2t)} \sim \frac{0}{0}$

$$\lim_{t \rightarrow 0} \frac{\tan(6t)}{\sin(2t)}$$

$$= \lim_{t \rightarrow 0} \frac{\left( \frac{\sin(6t)}{\cos(6t)} \right)}{\sin(2t)}$$

$$= \lim_{t \rightarrow 0} \frac{\sin(6t)}{6t} \cdot \frac{1}{\left( \frac{\sin(2t)}{2t} \right)} \cdot \frac{1}{\cos(6t)} \cdot \frac{6t}{2t}$$

$$= 1 \cdot \frac{1}{1} \cdot 1 \cdot 3$$

$$= 3$$