Question 1: Find the derivatives of the following functions (you do not have to simplify your answers):

(a)[3 points] $f(x) = -2x^5 + 5\pi^2 - \dfrac{1}{x^2}$

(b)[3 points] $y = \sqrt{\dfrac{1}{t^2 - 7}}$

(c)[4 points] $g(x) = 3x - 5\cos^2(\pi x)$

Question 2: Again, with these questions you do not have to simplify your answers:

(a)[3 points] Find $f'(x)$ if $f(x) = \dfrac{\sqrt{x+1}}{x^2+1}$.

(b)[3 points] Find y' if $y = \cos(\sin(x^2 + x))$

(c)[4 points] y is defined implicitly as a function of x by the equation $x \sin y = y \cos x$. Find $\dfrac{dy}{dx}$.

Question 3:

(a)[**3 points**] Compute $\int \frac{\sqrt{x}}{2} - 3\sin x \, dx$.

(b)[**3 points**] A bug walking along the x-axis has position given as a function of time $x(t) = 2t^3 - 15t^2 + 24t$, where t is in seconds. What is the acceleration of the bug at time $t = 2$ seconds?

(c)[**4 points**] Solve the differential equation $y' = t^{3/2} - t - 1$ where $y = 0$ when $t = 0$.

Question 4:

(a)[5 points] Find the equation of the tangent line to the curve $y = \dfrac{1+\sin x}{1-\sin x}$ at the point where $x = \pi$.

(b)[5 points] There are two points on the curve $y = x^2 + x$ at which the tangent lines to the curve also pass through $(1, 0)$. Find the x coordinates of these points.

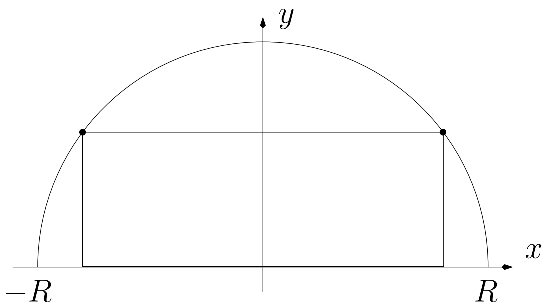
Question 5:

(a)[**7 points**] Let $f(x) = (1 + x)\sin(x)$. Use a tangent line approximation to estimate $f(0.1)$. Round your answer to 2 decimal places.

(b)[**3 points**] Is your estimate in part (a) an over-estimate or under-estimate? Explain using calculus.

Question 6: [10 points]
Sand is falling onto a pile at a rate of 1 m^3 per minute. The pile of sand maintains the shape of a cone with height equal to half the diameter of the base. How fast is the height of the sand pile increasing when the height is exactly 3 m? Give units in your answer.

Question 7: [10 points]
Find the area of the largest rectangle that can be inscribed in a semicircle of radius R if one side of the rectangle lies along the diameter of the semicircle.



Question 8: [10 points]
A cylindrical container open at the top is to be made with 300π m^2 of material. Find the radius and height which produce the container of greatest volume. Be sure to justify that your answer does indeed give the maximum.

Question 9:

(a)[**5 points**] You are using Newton's Method to locate a root of the equation $f(x) = 0$, and you make an initial guess of $x_1 = 2$ for the root. The tangent line to the curve $y = f(x)$ at $x = 2$ has equation $3y = 10x - 19$. What will be the next approximation x_2 obtained using Newton's Method?

(b)[**5 points**] The equation $x^2 = \dfrac{1}{\sin x}$ has one solution for $0 < x < 2$. Find x_2, the second approximation for the root obtained using Newton's Method. Round your answer to three decimal places.

Question 10: Consider the function $f(x) = \dfrac{x^3}{1+x}$.

(a)[**2 points**] Find the vertical asymptotes as well as the x and y intercepts of the graph of $y = f(x)$.

(b)[**3 points**] Find the intervals of increase and decrease of $f(x)$. State the x coordinate of any relative extrema.

continued on next page. . .

(c)[**3 points**] Find the intervals on which $f(x)$ is concave up, and the intervals on which $f(x)$ is concave down. State the x coordinate of any inflection points.

(d)[**2 points**] Use your results from (a), (b) and (c) to sketch the graph of $y = f(x)$.