

(1) [7 points] Determine the critical numbers of $F(x) = x^{4/5}(x-4)^2$.

$$\begin{aligned} F'(x) &= \frac{4}{5} x^{-\frac{1}{5}} (x-4)^2 + x^{\frac{4}{5}} \cdot 2(x-4) \\ &= \frac{4(x-4)^2}{5x^{\frac{1}{5}}} + \frac{2x^{\frac{4}{5}}(x-4)}{5x^{\frac{1}{5}}} \cdot 5x^{\frac{1}{5}} \\ &= \frac{4(x-4)^2 + 10x(x-4)}{5x^{\frac{1}{5}}} \\ &= \frac{(x-4)[4(x-4) + 10x]}{5x^{\frac{1}{5}}} \\ &= \frac{(x-4)(14x-16)}{5x^{\frac{1}{5}}} \end{aligned}$$

• $F'(x) = 0$: $(x-4)(14x-16) = 0$
 $\Rightarrow x-4=0$, $14x-16=0$
 $x=4$, $x = \frac{16}{14} = \frac{8}{7}$

• $F'(x)$ does not exist at $x=0$

\therefore critical numbers are $x=0, \frac{8}{7}, 4$

(2) [8 points] Find the absolute maximum and absolute minimum values of

$$f(x) = 2x^3 - 3x^2 - 12x + 1$$

on the closed interval $[-2, 3]$.

$$\begin{aligned} f'(x) &= 6x^2 - 6x - 12 \\ &= 6(x^2 - x - 2) \\ &= 6(x-2)(x+1) \end{aligned}$$

$$\therefore f'(x) = 0 \text{ at } x=2, x=-1$$

($f'(x)$ exists for all real x).

x	$f(x) = 2x^3 - 3x^2 - 12x + 1$
-2	-3
-1	8
2	-19
3	-8

$\therefore f$ has an abs. max. of 8 at $x = -1$,

f has an abs. min. of -19 at $x = 2$.