Math 121 S09N02 - Quiz 5
Mar 13 2009

Name: SOLUTIONS
Stu#:

(1) [**5 points**] Differentiate

$$y = \frac{x}{1 - \ln(x-1)}$$

$$y' = \frac{1 - \ln(x-1) - x\left[-\frac{1}{x-1}\right]}{\left[1 - \ln(x-1)\right]^2}$$

$$= \frac{1 - \ln(x-1) + \frac{x}{x-1}}{\left[1 - \ln(x-1)\right]^2}$$

(2) [**5 points**] Use logarithmic differentiation to find the derivative:

$$y = \frac{\sin^2 x \, \tan^4 x}{(x^2+1)^2}$$

$$\ln y = 2\ln(\sin x) + 4\ln(\tan x) - 2\ln(x^2+1)$$

$$\frac{1}{y} y' = \frac{2}{\sin x} \cdot \cos x + \frac{4}{\tan x} \cdot \sec^2 x - \frac{2}{x^2+1} \cdot (2x)$$

$$\therefore y' = \frac{\sin^2 x \tan^4 x}{(x^2+1)^2}\left[\frac{2\cos x}{\sin x} + \frac{4\sec^2 x}{\tan x} - \frac{4x}{x^2+1}\right]$$

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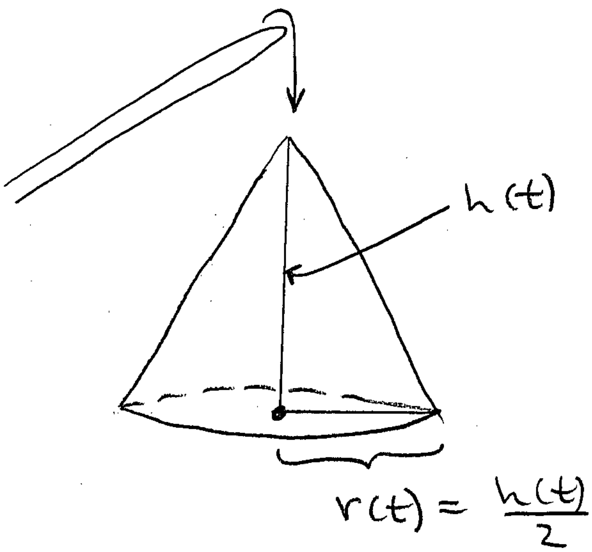
(3) [**5 points**] Gravel is being dumped from a conveyor belt at a rate of 10 m^3/min to form a pile in the shape of a cone. The cone-shaped pile of gravel grows in such a way that the height is always equal to the diameter of the base. How fast is the height of the pile increasing when the pile is 2 m tall? State units with your answer. (Recall that the volume of a cone of base radius r and height h is $V = \pi r^2 h/3$.)



$V(t)$ = volume of cone at time t minutes

$$\frac{dV}{dt} = 10 \frac{m^3}{min}.$$

Find $\frac{dh}{dt}$ when $h = 2m$.

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12} h^3$$

$$\therefore \frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \frac{dh}{dt}$$

When $h = 2$ m: $10 = \frac{\pi}{\cancel{12}\,4} \cdot \cancel{3}(\cancel{2})^{2} \cdot \frac{dh}{dt}$

$$\therefore \frac{dh}{dt} = \frac{10}{\pi} \frac{m}{min}$$

$\therefore$ height is increasing at $\frac{10}{\pi} \frac{m}{min}$ when $h = 2m$.