

(1) [5 points] Evaluate the limit, if it exists:

$$\lim_{h \rightarrow 0} \frac{(4+h)^2 - 16}{h} \sim \frac{0}{0}$$

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{(4+h)^2 - 16}{h} \\ &= \lim_{h \rightarrow 0} \frac{16 + 8h + h^2 - 16}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(8+h)}{h} \\ &= 8 \end{aligned}$$

(2) [5 points] Use the Squeeze Theorem to show that  $\lim_{x \rightarrow 0} x^4 \cos\left(\frac{2}{x}\right) = 0$ .

$$-x^4 \leq x^4 \cos\left(\frac{2}{x}\right) \leq x^4$$

Since  $\lim_{x \rightarrow 0} (-x^4) = 0 = \lim_{x \rightarrow 0} x^4$ , by the Squeeze Theorem,

$$\lim_{x \rightarrow 0} x^4 \cos\left(\frac{2}{x}\right) = 0.$$

(3) [5 points] Find the limit:  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta + \tan \theta} \sim \frac{0}{0}$

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta + \tan \theta}$$

$$= \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\left(\theta + \frac{\sin \theta}{\cos \theta}\right) \div \theta}$$

$$= \lim_{\theta \rightarrow 0} \frac{\frac{\sin \theta}{\theta}}{\frac{\theta}{\theta} + \frac{\sin \theta}{\theta} \cdot \frac{1}{\cos \theta}}$$

$$= \frac{1}{1 + 1}$$

$$= \frac{1}{2}$$