Question 1:

(a)[5 points] Use a linear approximation to estimate $\sqrt{102}$.

$$f(x) = x^{1/2}, \quad a = 100, \quad f(a) = f(100) = 10$$

$$f'(x) = \frac{1}{2} x^{-\frac{1}{2}} \; ; \; f'(a) = f'(100) = \frac{1}{2} \frac{1}{\sqrt{100}} = \frac{1}{20}$$

$$\therefore L(x) = f(a) + f'(a)(x-a)$$

$$= 10 + \frac{1}{20}(x-100)$$

$$\therefore \sqrt{102} = f(102) \simeq L(102) = 10 + \frac{1}{20}(102-100)$$

$$= 10 + \frac{1}{10}$$

$$= \boxed{10.1}$$

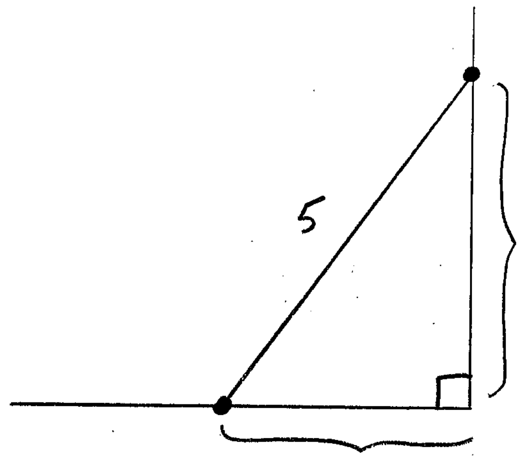
(b)[5 points] Let $f(x) = \dfrac{\ln(1+x^2)}{\cos(\pi x)}$. Compute $f'(0)$.

$$f'(x) = \frac{\cos(\pi x)\left(\frac{1}{1+x^2}\right)(2x) - \ln(1+x^2)\left(-\sin(\pi x)\right)\cdot \pi}{\left(\cos(\pi x)\right)^2}$$

$$\therefore f'(0) = \frac{\cancel{\cos(0)\left(\frac{1}{1+0}\right)\cdot 2\cdot 0} \to 0 - \cancel{\ln(1+0)\left(-\sin(0)\right)\cdot \pi} \to 0}{\cancel{\left(\cos(0)\right)^2} \to 1}$$

$$= 0$$

Question 2 [10 points]:
The top of a 5 m long ladder leans against a vertical wall. The top of the ladder slides down the wall as the bottom is pulled horizontally away from the wall at 1/3 m/s. At what rate is the top of the ladder sliding down the wall when the top of the ladder is 3 m above the ground?



$y(t)$, $\frac{dy}{dt} = ?$

$x(t)$, $\frac{dx}{dt} = \frac{1}{3}\ \frac{m}{s}$.

Find $\frac{dy}{dt}$ when $y = 3$ m.

$$x^2 + y^2 = 5^2 \implies y = (25 - x^2)^{\frac{1}{2}}$$

$$\frac{dy}{dt} = \frac{1}{2}(25 - x^2)^{-\frac{1}{2}}(-2x)\frac{dx}{dt}$$

When $y = 3$, $x = \sqrt{5^2 - 3^2} = 4$

$$\therefore \frac{dy}{dt} = \frac{1}{\cancel{2}}(25 - 4^2)^{-\frac{1}{2}}(-\cancel{2} \cdot 4)\left(\frac{1}{3}\right)$$

$$= \boxed{\frac{-4}{9}\ \frac{m}{s}}$$

$\therefore$ top of ladder is sliding down the wall at $\frac{4}{9}\ \frac{m}{s}$.

Question 3 [10 points]:
Determine the absolute maximum and absolute minimum values of $\underbrace{f(x) = e^{x^2-4x+4}}_{\text{continuous}}$ on $\underbrace{[1, 4]}_{\text{closed.}}$.

$$f'(x) = e^{x^2-4x+4}[2x-4]$$

$$f'(x) = 0 \Rightarrow 2x - 4 = 0$$

$$\Rightarrow x = 2$$

x	$f(x) = e^{x^2-4x+4}$
1	$e^1 = e$
2	$e^0 = 1$ ← abs. min
4	e^4 ← abs. max.

$\therefore$ f has an abs. min. of 1 at $x = 2$
f has an abs. max. of e^4 at $x = 4$

Question 4:

(a)[8 points] Determine the intervals of increase and decrease of $f(x) = (x^2 - 1)^3$.

$$f'(x) = 3(x^2-1)^2(2x) = 6x(x^2-1)^2$$

$f'(x) = 0$ at $x = 0, 1, -1$

($f'(x)$ exists for all real x.)

	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
$f'(x) = 6x(x^2-1)^2$	$-$	0	$-$	0	$+$	0	$+$
$f(x) = (x^2-1)^3$	$\searrow$	0	$\searrow$	-1	$\nearrow$	0	$\nearrow$

$\therefore$ f is increasing on $(0,1)$ & $(1,\infty)$

f is decreasing on $(-\infty,-1)$ & $(-1,0)$

(b)[2 points] Use your results in part (a) to find all local extreme values of $f(x) = (x^2 - 1)^3$.

f has a local minimum of $y = -1$ at $x = 0$;

(f has no local maxima)

Question 5:

(a) **[5 points]** The function $f(x) = x^4 e^x$ has second derivative $f''(x) = e^x x^2 (x+6)(x+2)$. Determine the intervals of concavity of $f(x)$. It is not necessary to determine the inflection points.

$f''(x) = 0$ at $x = 0, -6, -2$

	-7	-6	-3	-2	-1	0	1
$f''(x) = e^x x^2 (x+6)(x+2)$:	$+$		$-$		$+$		$+$
$f(x)$:	$\cup$		$\cap$		$\cup$		$\cup$

$\therefore$ Graph of $y = f(x)$ is concave down on $(-6,-2)$, concave up on $(-\infty,-6)$, $(-2,0)$ & $(0,\infty)$.

(b) **[5 points]** Use logarithmic differentiation to determine y' if $y = (\sin x)^{\ln x}$.

$$y = (\sin x)^{\ln x}$$

$$\ln y = (\ln x)\ln(\sin x)$$

$$\frac{1}{y} y' = \frac{1}{x} \cdot \ln(\sin x) + \ln x \cdot \frac{1}{\sin x} \cdot \cos x$$

$$\therefore\ y' = (\sin x)^{\ln x}\left[\frac{1}{x}\ln(\sin x) + \ln x \cdot \frac{\cos x}{\sin x}\right]$$