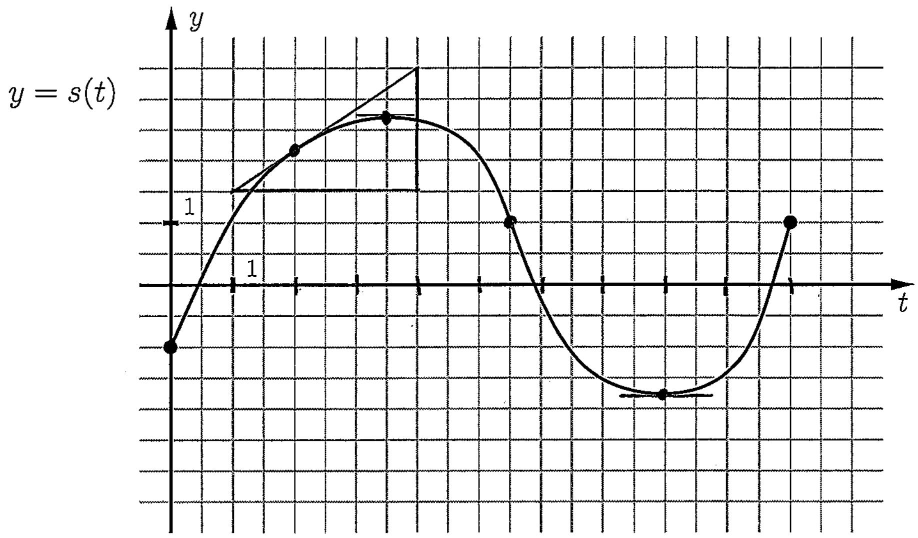
Question 1:

(a) The following is the graph of $y = s(t)$, the displacement of a particle in metres at time t seconds.



(i) [**2 points**] Estimate the velocity at $t = 2$ seconds. State units.

$$v(2) = s'(2) \approx \frac{4}{6} = \frac{2}{3}\ \frac{m}{s}$$

(ii) [**2 points**] When is the particle at rest?

$t = 3.5$ s & $t = 8$ s (tangent lines are horizontal.)

(iii) [**2 points**] At what time did the velocity change from decreasing to increasing?

$t = 5.5$ s

(b) [**4 points**] The equation of motion of a particle is $s(t) = 2t^3 - 3t^2 + 7$ where s is in metres and t in seconds. Determine the time at which acceleration is zero.

$$v(t) = s'(t) = 6t^2 - 6t$$

$$a(t) = v'(t) = 12t - 6$$

$$\therefore \quad a(t) = 0 \Rightarrow 12t - 6 = 0$$

$$\Rightarrow t = \frac{6}{12} = \boxed{\frac{1}{2}\ s}$$

Question 2:

(a) [3 points] Differentiate: $y = \dfrac{\cos(x)}{1+x^2}$

$$y' = \frac{(1+x^2)(-\sin(x)) - \cos(x)(2x)}{(1+x^2)^2}$$

(b) [3 points] Differentiate: $f(t) = 2\sqrt{t}\sec(t)$ $= 2t^{\frac{1}{2}}\sec(t)$

$$f'(t) = 2\left[\frac{1}{2}t^{-\frac{1}{2}}\sec(t) + t^{\frac{1}{2}}\sec(t)\tan(t)\right]$$

$$= \frac{\sec(t)}{\sqrt{t}} + 2\sqrt{t}\sec(t)\tan(t)$$

(c) [4 points] Find $\dfrac{dy}{dx}$: $y = \dfrac{4x^2e^x}{x^2+\pi^2}$

$$\frac{dy}{dx} = \frac{(x^2+\pi^2)\frac{d}{dx}(4x^2e^x) - 4x^2e^x\frac{d}{dx}(x^2+\pi^2)}{(x^2+\pi^2)^2}$$

$$= \frac{(x^2+\pi^2)\left[8xe^x + 4x^2e^x\right] - 4x^2e^x(2x)}{(x^2+\pi^2)^2}$$

Question 3:

(a)[3 points] Compute $g'(1)$: $g(r) = 3e^r\sqrt[3]{8r}$ $= 3e^r \cdot 2 \cdot r^{\frac{1}{3}} = 6e^r r^{\frac{1}{3}}$

$$g'(r) = 6e^r r^{\frac{1}{3}} + 6e^r \cdot \frac{1}{3} r^{-\frac{2}{3}}$$

$$\therefore g'(1) = 6e^1 \cdot 1^{\frac{1}{3}} + 2e^1 \cdot 1^{-\frac{2}{3}}$$

$$= 8e$$

(b)[3 points] Differentiate: $y = \left(x + \frac{1}{x^2}\right)^{-5}$ $= (x + x^{-2})^{-5}$

$$y' = -5(x + x^{-2})^{-6} \cdot (1 - 2x^{-3})$$

$$= -5\left(x + \frac{1}{x^2}\right)^{-6} \cdot \left(1 - \frac{2}{x^3}\right)$$

(c)[4 points] Differentiate: $q(t) = \sin(t\csc(t))$

$$q'(t) = \cos[t\csc(t)] \cdot [\csc(t) - t\csc(t)\cot(t)]$$

Question 4:

(a)[3 points] Find $\frac{dy}{dz}$: $y = \sqrt{\tan(\sqrt{z})}$ $= \left[\tan\left(z^{\frac{1}{2}}\right)\right]^{\frac{1}{2}}$

$$\frac{dy}{dz} = \frac{1}{2}\left[\tan\left(z^{\frac{1}{2}}\right)\right]^{-\frac{1}{2}} \cdot \sec^2\left(z^{\frac{1}{2}}\right) \cdot \frac{1}{2} z^{-\frac{1}{2}}$$

$$= \frac{1}{4} \frac{\sec^2(\sqrt{z})}{\sqrt{z \tan(\sqrt{z})}}$$

(b)[3 points] Find y': $y = e^{x^7 \cos x}$

$$y' = e^{x^7 \cos x}\left[7x^6 \cos x - x^7 \sin x\right]$$

(b)[4 points] Compute $f''(0)$: $f(x) = x^2 e^{x^3}$

$$f'(x) = 2x e^{x^3} + x^2 e^{x^3} \cdot 3x^2$$

$$= (2x + 3x^4) e^{x^3}$$

$$f''(x) = (2 + 12x^3) e^{x^3} + (2x + 3x^4) e^{x^3} \cdot 3x^2$$

$$\therefore f''(0) = (2+0)e^0 + (0+0)e^0 \cdot 0$$

$$= \boxed{2}$$

Question 5:

(a)[**5 points**] Evaluate the limit $\lim_{x\to\infty} \frac{2e^{5x}-e^{x}-1}{3e^{5x}+5e^{3x}}$ $\sim \frac{\infty}{\infty}$

$$= \lim_{x\to\infty} \frac{2e^{5x}-e^{x}-1 \div e^{5x}}{3e^{5x}+5e^{3x} \div e^{5x}}$$

$$= \lim_{x\to\infty} \frac{2-e^{-4x}-e^{-5x}}{3+5e^{-2x}}$$

$$= \frac{2}{3}$$

(b)[**5 points**] Find the equation of the tangent line to the curve $x^3+y^3=4xy+1$ at the point $(2,1)$.

$$x^3+y^3 = 4xy+1$$

$$\frac{d}{dx}\left[x^3+y^3\right] = \frac{d}{dx}\left[4xy+1\right]$$

$$3x^2+3y^2y' = 4\left[1\cdot y+xy'\right]$$

at $(2,1)$; $3(2)^2+3(1)^2y' = 4\left[1+2y'\right]$

$$12+3y' = 4+8y'$$

$$5y' = 8$$

$$y' = \frac{8}{5}$$

$\therefore$ $y-y_0 = m(x-x_0)$

$\boxed{y-1=\frac{8}{5}(x-2)}$ or $\boxed{y=\frac{8}{5}x-\frac{11}{5}}$