Math 121 S09N01 - Quiz 6 Name:
Mar 19 2009 Stu#: SOLUTIONS.

(1) [7 points] Determine the critical numbers of $f(x) = \dfrac{x-1}{x^2-x+1}$.

$$f'(x) = \frac{(x^2-x+1)(1) - (x-1)(2x-1)}{(x^2-x+1)^2}$$

$$= \frac{x^2-x+1 - 2x^2+3x-1}{(x^2-x+1)^2}$$

$$= \frac{-x^2+2x}{(x^2-x+1)^2}$$

$$= \frac{-x(x-2)}{(x^2-x+1)^2}$$

$f'(x) = 0$ at $x=0$, $x=2$

$f'(x)$ does not exist if $x^2-x+1=0$

$$\therefore x = \frac{-(-1) \pm \sqrt{(-1)^2-4(1)(1)}}{2(1)}$$

$$\left.= \frac{1 \pm \sqrt{-3}}{2}\right\} \therefore \text{ no real roots.}$$

$\therefore$ $x=0$ & $x=2$ are the only critical numbers.

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(2) [**8 points**] Find the absolute maximum and absolute minimum values of $f(t) = t\sqrt{4-t^2}$ on the closed interval $[-1,2]$.

$$f(t) = t(4-t^2)^{\frac{1}{2}}$$

$$f'(t) = (1)(4-t^2)^{\frac{1}{2}} + t\left(\tfrac{1}{\cancel{2}}\right)(4-t^2)^{-\frac{1}{2}}(-\cancel{2}t)$$

$$= (4-t^2)^{\frac{1}{2}} - \frac{t^2}{(4-t^2)^{\frac{1}{2}}}$$

$$= \frac{4-t^2-t^2}{(4-t^2)^{\frac{1}{2}}}$$

$$= \frac{2(2-t^2)}{(4-t^2)^{\frac{1}{2}}}$$

- $f'(t) = 0$: $2-t^2=0 \Rightarrow t=\sqrt{2}, \underbrace{-\sqrt{2}}_{\text{outside } [-1,2]}$

- $f'(t)$ does not exist at $t = 2, \underbrace{-2}_{\text{outside } [-1,2]}$

t	$f(t) = t\sqrt{4-t^2}$
-1	$(-1)\sqrt{4-(-1)^2} = -\sqrt{3}$
$\sqrt{2}$	$\sqrt{2}\sqrt{4-(\sqrt{2})^2} = 2$
2	$(2)\sqrt{4-2^2} = 0$

$\therefore$ f has an abs. max. of 2 at $t=\sqrt{2}$,
f has an abs. min. of $-\sqrt{3}$ at $t=-1$