Math 121 S09N01 - Quiz 5
Mar 13 2009

Name: SOLUTIONS
Stu#:

(1) [**5 points**] Differentiate

$$y = \ln\left(e^{-x} + xe^{-x}\right)$$

$$y' = \frac{1}{e^{-x} + xe^{-x}} \cdot \left[-e^{-x} + e^{-x} - xe^{-x}\right]$$

$$= \frac{-x\cancel{e^{-x}}}{\cancel{e^{-x}} + x\cancel{e^{-x}}}$$

$$= \frac{-x}{1+x}$$

(2) [**5 points**] Use logarithmic differentiation to find the derivative:

$$y = (\tan x)^{1/x}$$

$$\ln y = \frac{1}{x} \ln(\tan x)$$

$$\frac{1}{y} y' = \left(\frac{-1}{x^2}\right) \ln(\tan x) + \left(\frac{1}{x}\right) \frac{1}{\tan x} \cdot \sec^2 x$$

$$= -\frac{\ln(\tan x)}{x^2} + \frac{1}{x} \cdot \frac{\cos x}{\sin x} \cdot \frac{1}{\cos^2 x}$$

$$\therefore\ y' = (\tan x)^{\frac{1}{x}} \left[-\frac{\ln(\tan x)}{x^2} + \frac{\sec x \cdot \csc x}{x}\right]$$

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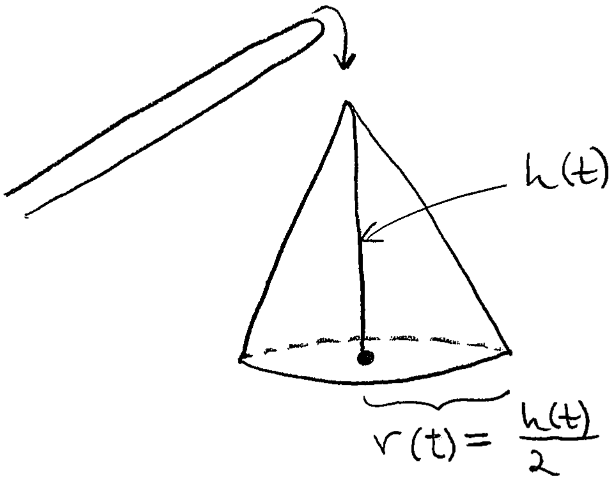
(3) [**5 points**] Gravel is being dumped from a conveyor belt at a rate of 10 m^3/min to form a pile in the shape of a cone. The cone-shaped pile of gravel grows in such a way that the height is always equal to the diameter of the base. How fast is the height of the pile increasing when the pile is 5 m tall? State units with your answer. (Recall that the volume of a cone of base radius r and height h is $V = \pi r^2 h/3$.)

$2r = h$

$\therefore\ r = \frac{h}{2}$



Let $V(t)$ = Volume of cone at time t minutes.

$$\frac{dV}{dt} = 10\ \frac{m^3}{min}$$

Find $\frac{dh}{dt}$ when $h = 5$ m.

$$\therefore\ V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12} h^3$$

$$\therefore\ \frac{dV}{dt} = \frac{\pi}{12} 3h^2 \frac{dh}{dt}$$

when $h = 5$ m: $10 = \frac{\pi}{\cancel{12}\,4} \cdot \cancel{3}(5^2)\frac{dh}{dt}$

$$\therefore\ \frac{dh}{dt} = \frac{40}{25\pi} = \frac{8}{5\pi}\ \frac{m}{min}$$

$\therefore$ height is increasing at $\frac{8}{5\pi}\ \frac{m}{min}$ when $h = 5$ m.