Question 1:

(a)[7 points] Solve for x:

$$5^{x^2-3} = 25^x$$

$$5^{x^2-3} = (5^2)^x$$

$$5^{x^2-3} = 5^{2x}$$

$$\therefore \; x^2-3 = 2x$$

$$x^2-2x-3 = 0$$

$$(x-3)(x+1) = 0$$

$$\boxed{x=3, \quad x=-1}$$

(b)[3 points] Find the exact value of $\log_9(1/3)$

Let $x = \log_9\left(\frac{1}{3}\right)$, so $9^x = \frac{1}{3}$

$$\therefore (3^2)^x = 3^{-1}$$

$$3^{2x} = 3^{-1}$$

$$2x = -1$$

$$\boxed{x = -\frac{1}{2}}$$

Question 2:

(a)[3 points] Write as a single logarithm:

$$\ln\left(\frac{x}{y}\right) - 2\ln x^3 - 4\ln y$$

$$= \ln\left(\frac{x}{y}\right) - \ln(x^3)^2 - \ln y^4$$

$$= \ln\left(\frac{x}{y}\right) - \ln(x^6) - \ln y^4$$

$$= \ln\left(\frac{x/y}{x^6 y^4}\right)$$

$$= \boxed{\ln\left(\frac{1}{x^5 y^5}\right)}$$

(b)[7 points] Solve for x:

$$\ln x = \ln 10 - \ln(x-3)$$

$$\ln x + \ln(x-3) = \ln 10$$

$$\ln[x(x-3)] = \ln 10$$

$$x(x-3) = 10$$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$x = 5, -2$$

$\boxed{\therefore\ x = 5 \text{ is the only solution}}$

Check: $x=5$:

$$\ln 5 \;\;\; \ln 10 - \ln(5-3)$$

$$\ln 5 \;\;\; \ln 10 - \ln(2)$$

$$\ln 5 \;\;\; \ln\left(\frac{10}{2}\right)$$

$$\ln 5 = \ln 5 \checkmark$$

$x = -2$: $\ln(-2)$ | $\ln(10) - \ln(-2-3)$

not defined.

$\therefore\ x = -2$ is not a solution.

Question 3: A population grows according to the model $P(t) = P_0 e^{kt}$ where P_0 is the population at time $t = 0$ and time is measured in years.

(a)[**5 points**] If the initial population doubles in 10 years, how long does it take to triple?

$$2\cancel{P_0} = \cancel{P_0}\, e^{k \cdot 10}$$

$$\ln 2 = k \cdot 10$$

$$k = \frac{\ln 2}{10}$$

Now solve $P(t) = 3P_0$ for t:

$$3\cancel{P_0} = \cancel{P_0}\, e^{\left(\frac{\ln 2}{10}\right)t}$$

$$\ln 3 = \left(\frac{\ln 2}{10}\right) t$$

$$\therefore\ t = \frac{\ln 3}{\left(\frac{\ln 2}{10}\right)} = \frac{10 \ln 3}{\ln 2} \doteq \boxed{15.8 \text{ yrs.}}$$

(b)[**5 points**] Again, if the initial population doubles in 10 years, and the population is 5000 after 2 years, what was the initial population?

Again $k = \frac{\ln 2}{10}$.

Solve $5000 = P_0\, e^{\left(\frac{\ln 2}{10}\right)\cdot 2}$ for P_0:

$$P_0 = \frac{5000}{e^{\left(\frac{\ln 2}{10}\right)\cdot 2}} \doteq \boxed{4353 \text{ individuals}}$$

Question 4: Let

$$\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 4 & 3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 3 & 1 \\ -1 & -4 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 2 \\ -2 \end{bmatrix},$$

(a)[4 points] Compute $\frac{1}{2}(2\mathbf{A} - 3\mathbf{B})\mathbf{C}$

$$\frac{1}{2}(2A-3B)C = \frac{1}{2}\left(2\begin{bmatrix} 1 & -1 \\ 4 & 3 \end{bmatrix} - 3\begin{bmatrix} 3 & 1 \\ -1 & -4 \end{bmatrix}\right)\begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -7 & -5 \\ 11 & 18 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & -5 \\ 11 & 18 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ -7 \end{bmatrix}$$

(b)[4 points] Compute $\mathbf{C}^T\mathbf{A}\mathbf{C}$

$$C^T A C = \begin{bmatrix} 2 & -2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -8 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \end{bmatrix}$$

(b)[2 points] If $\mathbf{P}$ has size 4×3, $\mathbf{Q}$ has size 3×2 and $\mathbf{R}$ has size 2×5, what is the size of $\mathbf{PQR}$?

$$\left. P_{4\times3}\, Q_{3\times2}\, R_{2\times5} \right\} \therefore \text{ PQR has size } 4\times5$$

Question 5 [10 points]: Solve the following system of equations using matrix reduction:

$$\begin{aligned} 2x + 2z &= 2 \\ 2x + y + 4z &= 9 \\ -4x + z &= 11 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 2 & 2 \\ 2 & 1 & 4 & 9 \\ -4 & 0 & 1 & 11 \end{array}\right]$$

$\frac{1}{2}R_1$: $$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 2 & 1 & 4 & 9 \\ -4 & 0 & 1 & 11 \end{array}\right]$$

$(-2)R_1 + R_2$:
$4R_1 + R_3$: $$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 5 & 15 \end{array}\right]$$

$\frac{1}{5}R_3$: $$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$(-1)R_3 + R_1$:
$(-2)R_3 + R_2$: $$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\boxed{\begin{aligned} \therefore\ x &= -2 \\ y &= 1 \\ z &= 3 \end{aligned}}$$