Question 1:

(a)[7 points] Solve for x:

$$5^{x^2-3} = 25^x$$

(b)[3 points] Find the exact value of $\log_9(1/3)$

Question 2:

(a)[3 points] Write as a single logarithm:

$$\ln\left(\frac{x}{y}\right) - 2\ln x^3 - 4\ln y$$

(b)[7 points] Solve for x:

$$\ln x = \ln 10 - \ln(x-3)$$

Question 3: A population grows according to the model $P(t) = P_0 e^{kt}$ where P_0 is the population at time $t = 0$ and time is measured in years.

(a)[5 points] If the initial population doubles in 10 years, how long does it take to triple?

(b)[5 points] Again, if the initial population doubles in 10 years, and the population is 5000 after 2 years, what was the initial population?

Question 4: Let

$$\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 4 & 3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 3 & 1 \\ -1 & -4 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 2 \\ -2 \end{bmatrix},$$

(a)[4 points] Compute $\frac{1}{2}(2\mathbf{A} - 3\mathbf{B})\mathbf{C}$

(b)[4 points] Compute $\mathbf{C}^{\mathrm{T}}\mathbf{AC}$

(b)[2 points] If $\mathbf{P}$ has size 4×3, $\mathbf{Q}$ has size 3×2 and $\mathbf{R}$ has size 2×5, what is the size of $\mathbf{PQR}$?

Question 5 [10 points]: Solve the following system of equations using matrix reduction:

$$\begin{aligned} 2x + 2z &= 2 \\ 2x + y + 4z &= 9 \\ -4x + z &= 11 \end{aligned}$$

You may find some of the following formulas useful:

$$\sin^2(A) + \cos^2(A) = 1$$

$$\tan^2(A) + 1 = \sec^2(A)$$

$$1 + \cot^2(A) = \csc^2(A)$$

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

$$\sin(2A) = 2\sin(A)\cos(A)$$

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

$$\cos(2A) = 1 - 2\sin^2(A)$$

$$\cos(2A) = 2\cos^2(A) - 1$$

$$\sin^2(A/2) = \frac{1-\cos(A)}{2} \qquad \cos^2(A/2) = \frac{1+\cos(A)}{2}$$

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a_n = a + (n-1)d \qquad a_n = ar^{n-1}$$

$$S_n = n\frac{(a_1 + a_n)}{2} \qquad S_n = \frac{n[2a + (n-1)d]}{2} \qquad S_n = \frac{a(1-r^n)}{1-r}$$