Question 1:

(a)[5 points] Find the exact value of $\cos\left(-\frac{5\pi}{12}\right)$. (Hint: $-\frac{5}{12} = \frac{3}{12} - \frac{8}{12}$.)

$$\cos\left(-\frac{5\pi}{12}\right) = \cos\left(\frac{3\pi}{12} - \frac{8\pi}{12}\right)$$

$$= \cos\left(\frac{\pi}{4} - \frac{2\pi}{3}\right)$$

$$= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{2\pi}{3}\right) + \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{2\pi}{3}\right)$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \boxed{\frac{\sqrt{2}\,(\sqrt{3}-1)}{4}}$$

(b)[5 points] Find the exact value of $\sin\left(\frac{\pi}{12}\right)$. (Hint: $\frac{\pi}{12} = \frac{(\pi/6)}{2}$.)

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{(\pi/6)}{2}\right)$$

$$= \sqrt{\sin^2\left(\frac{(\pi/6)}{2}\right)}$$

note: positive square root since $\frac{\pi}{12}$ is in first quadrant.

$$= \sqrt{\frac{1-\cos\left(\frac{\pi}{6}\right)}{2}}$$

$$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$$

$$= \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$$

Question 2 [10 points]: Find all solutions $0 \leq x < 2\pi$ to

$$2\cos^2 x + 3\cos x + 1 = 0\ .$$

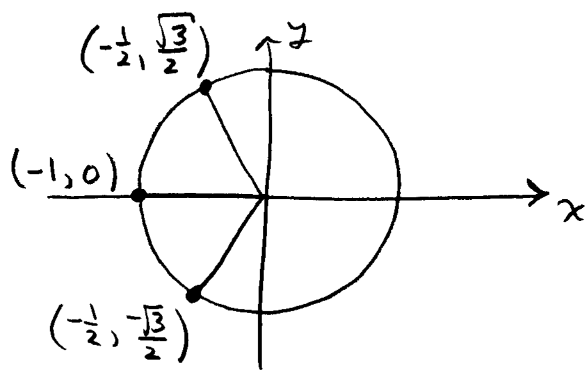
Letting $w = \cos x$, solve $2w^2 + 3w + 1 = 0$

$$\therefore\ w = \frac{-3 \pm \sqrt{9 - 4(2)(1)}}{2(2)} = \frac{-3 \pm 1}{4} = -\frac{1}{2},\ -1$$

$\therefore\ \cos x = -\frac{1}{2}$, $\cos x = -1$

$\therefore\ x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $x = \pi$

$\therefore\ \boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}, \pi}$



Question 3:

(a)[3 points] Simplify without using a calculator: $\cos\left(\sin^{-1}(1/3)\right)$.

Let $\theta = \sin^{-1}\left(\frac{1}{3}\right)$, so $\sin\theta = \frac{1}{3}$ where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$$\therefore \cos\left(\sin^{-1}\left(\tfrac{1}{3}\right)\right) = \cos\theta$$

$$= \sqrt{1-\sin^2\theta}$$

} positive square root since $\cos\theta \ge 0$ for $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$$= \sqrt{1-\left(\tfrac{1}{3}\right)^2}$$

$$= \sqrt{\tfrac{8}{9}}$$

$$= \boxed{\frac{2\sqrt{2}}{3}}$$

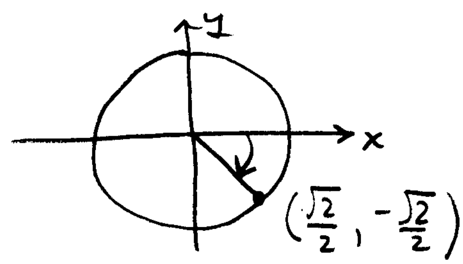
(b)[3 points] Simplify without using a calculator: $\arctan(-1)$.

$\arctan(-1)$ is the angle $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ such that $\tan(\theta) = -1$,

i.e. $\frac{\sin(\theta)}{\cos(\theta)} = -1$,

$\sin(\theta) = -\cos(\theta)$

$$\therefore \boxed{\theta = -\frac{\pi}{4}}$$



(c)[4 points] Simplify to a fraction containing no trigonometric functions: $\tan(\arccos(x))$.

Let $\theta = \arccos(x)$, so $\cos\theta = x$ where $0 \le \theta \le \pi$.

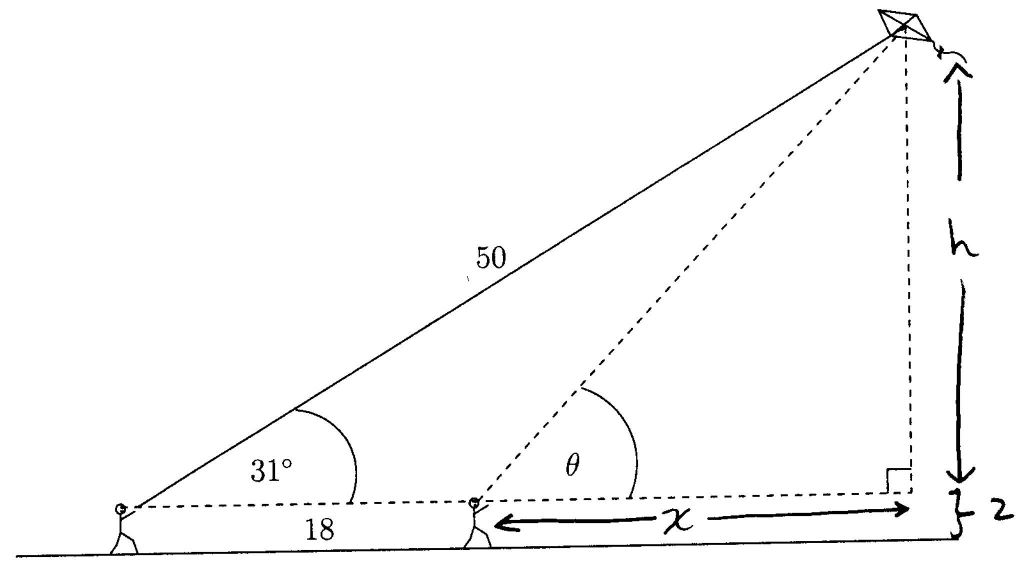
$$\therefore \tan(\arccos(x)) = \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$$

} positive square root since $\sin\theta \ge 0$ for $0 \le \theta \le \pi$

$$= \boxed{\frac{\sqrt{1-x^2}}{x}}$$

Question 4: Two friends standing 18 m apart on level ground are flying a kite. The kite, at the end of 50 m of string, is observed by the person holding the string to be at an angle of elevation of 31°.



(a)[5 points] If the friends are each 2 m tall, how high is the kite above the ground?

$\sin(31°) = \frac{h}{50}$

$\therefore \quad h = 50 \sin(31°)$

$\therefore$ height of kite above ground is

$h + 2 = 50\sin(31°) + 2$

$\doteq \boxed{27.8 \text{ m}}$

(b)[5 points] At what angle of elevation θ does the second friend see the kite?

$(18+x)^2 + h^2 = 50^2$ by Pythagoras,

$\therefore \quad x = \sqrt{50^2 - h^2} - 18$

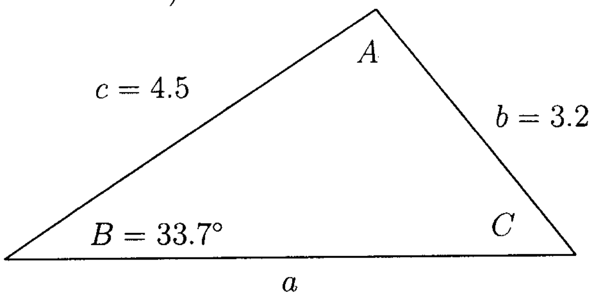
$\tan\theta = \frac{h}{x}$

$\therefore \theta = \arctan\left(\frac{h}{x}\right) = \arctan\left(\frac{50\sin(31°)}{\sqrt{50^2 - (50\sin(31°))^2} - 18}\right)$

$\doteq \boxed{46°}$

Question 5:

(a)[5 points] Angle C in the following triangle is acute. Solve the triangle (round final answers to 1 decimal):



$$\frac{\sin C}{4.5} = \frac{\sin(33.7)}{3.2}$$

$$\therefore\ C = \sin^{-1}\left[\frac{4.5\sin(33.7)}{3.2}\right]$$

$$\boxed{C \doteq 51.283 \doteq 51.3°}$$

$$\therefore\ A \doteq 180 - 33.7 - 51.283$$

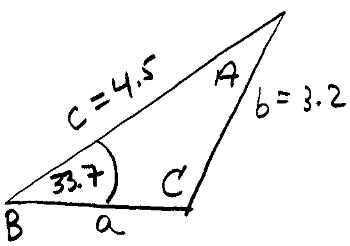
$$\boxed{A \doteq 95.017 \doteq 95.0°}$$

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$a \doteq \sqrt{(3.2)^2 + (4.5)^2 - 2(3.2)(4.5)\cos(95.017)}$$

$$\boxed{a \doteq 5.7}$$

(b)[5 points] There is another triangle with $B = 33.7°$, $b = 3.2$ and $c = 4.5$. Sketch and solve it (round final answers to 1 decimal).



using part (a),

$$C \doteq 180 - 51.283 \doteq 128.717$$

$$\boxed{C \doteq 128.7°}$$

$$A \doteq 180 - 33.7 - 128.717 \doteq 17.583$$

$$\boxed{A \doteq 17.6}$$

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$a \doteq \sqrt{(3.2)^2 + (4.5)^2 - 2(3.2)(4.5)\cos(17.583)}$$

$$\boxed{a \doteq 1.7}$$