Question 1:

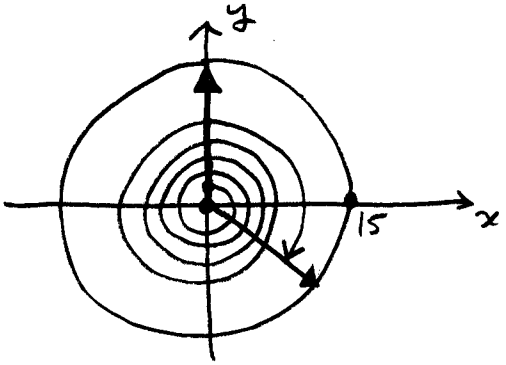
(a)[3 points] Convert $5\pi/12$ radians to degrees.

$$\left(\frac{5\pi}{12}\right)\left(\frac{180}{\pi}\right) = \boxed{75^\circ}$$

(b)[3 points] Convert $-215°$ to radians.

$$(-215^\circ)\left(\frac{\pi}{180^\circ}\right) = \boxed{-\frac{43\pi}{36}}$$

(c)[4 points] The minute hand of a clock is 15 cm long. How far (i.e. through what length of arc) does the tip of the minute hand travel in 4 hours and 20 minutes?



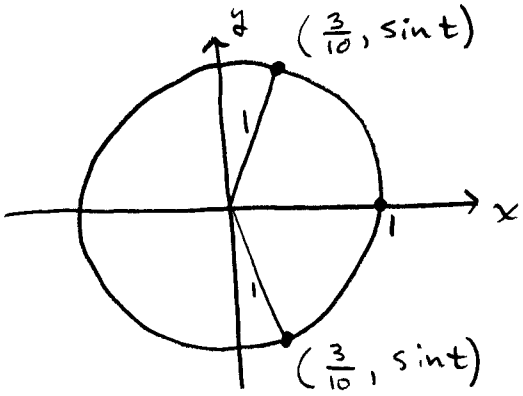
minute hand rotates through an angle $\theta = (4)(2\pi) + \left(\frac{20}{60}\right)(2\pi) = \left(\frac{26}{3}\right)\pi$.

Since $r = 15$ cm, corresponding arc length is

$$s = r\theta$$
$$= (15)\left(\frac{26}{3}\right)\pi$$
$$= \boxed{130\pi \text{ cm}}.$$

Question 2:

(a)[3 points] If $\cos t = 3/10$ find all possible values of $\sin t$.



$\therefore \left(\frac{3}{10}\right)^2 + \sin^2 t = 1$

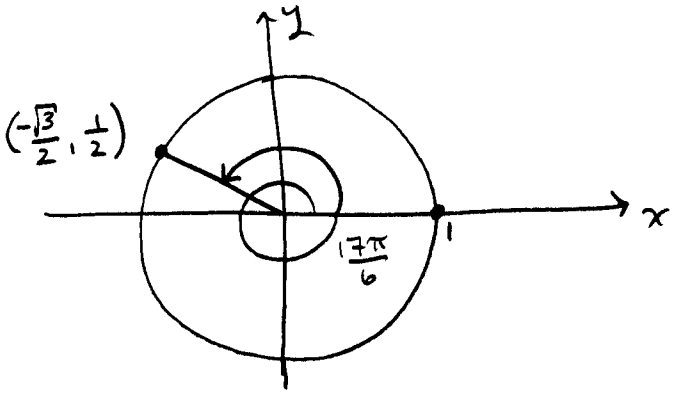
$\sin^2 t = 1 - \left(\frac{3}{10}\right)^2$

$= \frac{100}{100} - \frac{9}{100}$

$= \frac{91}{100}$

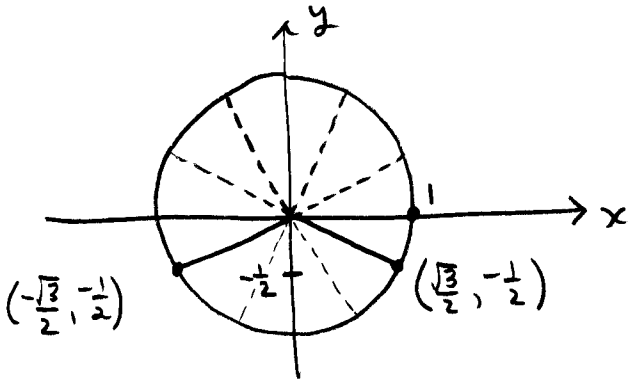
$\therefore \sin t = \frac{\sqrt{91}}{10}, -\frac{\sqrt{91}}{10}$

(b)[3 points] Find the exact value of $\sin(17\pi/6)$.



$\therefore \sin\left(\frac{17\pi}{6}\right) = \frac{1}{2}$

(c)[4 points] Find all angles θ such that $0 \le \theta < 2\pi$ and $\sin\theta = -1/2$.



$\therefore \sin\theta = -\frac{1}{2}$ at

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$.

Question 3:

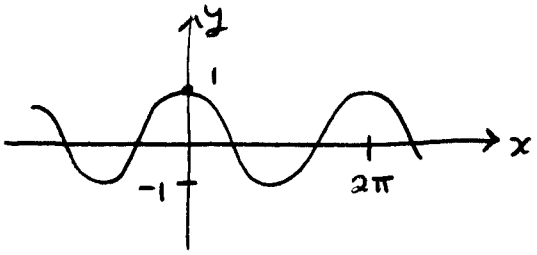
(a)[7 points] Carefully graph

$$y = -3\cos\left(2\pi x - \frac{\pi}{2}\right)$$

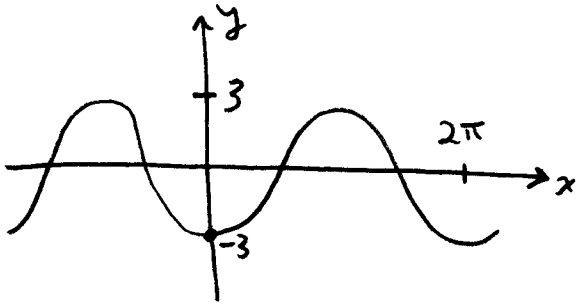
Indicate the scale on the x and y axes and label your graph.

$y = -3\cos\left(2\pi\left(x - \frac{1}{4}\right)\right)$

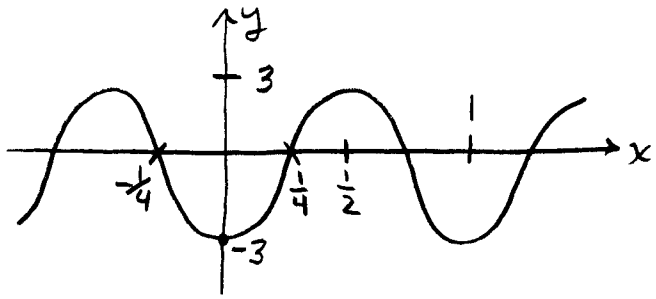
① $y = \cos(x)$



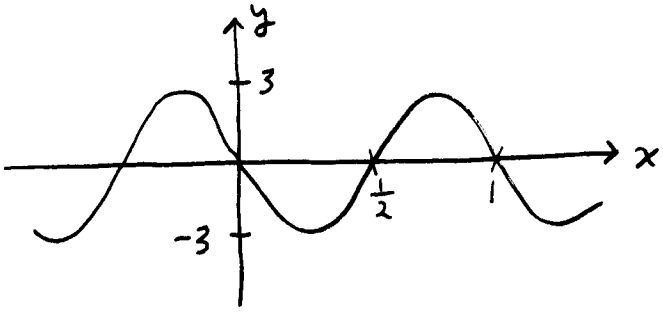
② $y = -3\cos(x)$



③ $y = -3\cos(2\pi x)$



④ $y = -3\cos\left(2\pi\left(x - \frac{1}{4}\right)\right)$



(b)[3 points] State the amplitude, period and phase-shift of the trigonometric function from part (a).

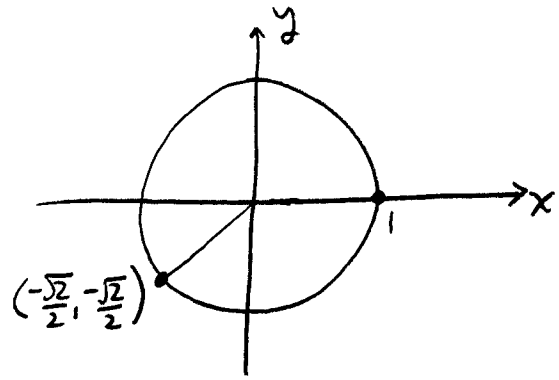
Amplitude = $|A| = |-3| = 3$

Period = $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$

Phase-Shift = $|C| = \frac{1}{4}$

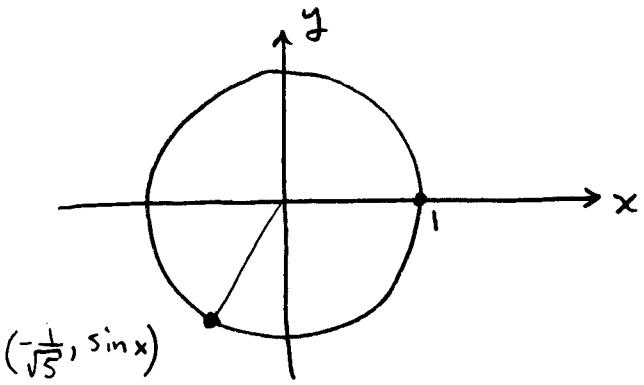
Question 4:

(a)[3 points] Find $\sec(29\pi/4)$.



$$\sec\left(\frac{29\pi}{4}\right) = \frac{1}{\cos\left(\frac{29\pi}{4}\right)} = \frac{1}{-\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}} = \boxed{-\sqrt{2}}.$$

(b)[4 points] If $\cos x = -1/\sqrt{5}$ where $\pi < x < 3\pi/2$, what is $\tan x$?



$$\tan x = \frac{\sin x}{\cos x} = \frac{-\sqrt{1-\cos^2 x}}{\cos x} = \frac{-\sqrt{1-\left(\frac{-1}{\sqrt{5}}\right)^2}}{-\frac{1}{\sqrt{5}}} = \frac{-\sqrt{\frac{4}{5}}}{-\frac{1}{\sqrt{5}}} = -\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{1} = \boxed{+2}$$

notice: negative square root since $\sin(x) < 0$ for $\pi < x < \frac{3\pi}{2}$.

(c)[3 points] Simplify

$$\frac{\cot\theta}{\cos\theta} - \csc\theta$$

$$= \frac{\left(\frac{\cos\theta}{\sin\theta}\right)}{\cos\theta} - \frac{1}{\sin\theta}$$

$$= \frac{\cos\theta}{\sin\theta\cos\theta} - \frac{1}{\sin\theta}$$

$$= \frac{1}{\sin\theta} - \frac{1}{\sin\theta}$$

$$= \boxed{0}$$

Question 5:

(a)[3 points] Find $\cos(393\pi)$

$$\cos(393\pi) = \cos(392\pi + \pi)$$
$$= \cos(\pi) \text{ by periodicity}$$
$$= \boxed{-1}$$

(b)[3 points] Find $\tan(405\pi/2)$

$$\tan\left(405\tfrac{\pi}{2}\right) = \frac{\sin\left(405\tfrac{\pi}{2}\right)}{\cos\left(405\tfrac{\pi}{2}\right)}$$
$$= \frac{\sin\left(202\pi + \tfrac{\pi}{2}\right)}{\cos\left(202\pi + \tfrac{\pi}{2}\right)}$$
$$= \frac{\sin\left(\tfrac{\pi}{2}\right)}{\cos\left(\tfrac{\pi}{2}\right)} \text{ by periodicity}$$

$\cos\left(\tfrac{\pi}{2}\right)$ ← zero!

$\therefore \tan\left(405\tfrac{\pi}{2}\right)$ is **not defined**

(c)[4 points] If $\sin(k\pi/3) < 0$ where k is a positive integer which is less than 5, what must be $\cos(k\pi/3)$?

Since $\sin\left(k\tfrac{\pi}{3}\right) < 0$ and $0 < k < 5$, $\tfrac{k\pi}{3}$ must have terminal side; ie. $k = 4$.

$\therefore \cos\left(\tfrac{k\pi}{3}\right) = -\tfrac{1}{2}$