(1)[8 points] The number of bacteria in a culture after t hours is given by $P(t) = P_0 e^{kt}$. After 3 hours it is observed that 400 bacteria are present. After 10 hours 2000 are present. What was the initial number of bacteria?

$$P(t) = P_0 e^{kt}$$

$$P(3) = P_0 e^{3k} = 400$$

$$P(10) = P_0 e^{10k} = 2000$$

$$\therefore \quad \frac{P_0 e^{10k}}{P_0 e^{3k}} = \frac{2000}{400}$$

$$e^{7k} = 5$$

$$\therefore \quad k = \frac{\ln 5}{7}$$

$$\therefore \quad P_0 e^{3\left(\frac{\ln 5}{7}\right)} = 400$$

$$P_0 = \frac{400}{e^{3\left(\frac{\ln 5}{7}\right)}} \doteq \boxed{201 \text{ bacteria}}$$

(2)[7 points] It was determined that 86% of the C-14 in a piece of charcoal had decayed through radioactivity. Determine the age of the piece of charcoal. (Recall that the half-life of C-14 is 5730 years, and that it decays according to the exponential model $A(t) = A_0 e^{kt}$.)

86% decayed, so 14% remains

$$A(t) = A_0 e^{kt}$$

Find k:

$$\frac{1}{2} A_0 = A_0 e^{5730k}$$

$$\therefore \ln\left(\frac{1}{2}\right) = 5730k$$

$$k = \frac{\ln(1/2)}{5730}.$$

Find t:

$$A(t) = A_0 e^{kt} = 0.14 A_0$$

$$\therefore A_0 e^{\left[\frac{\ln(\frac{1}{2})}{5730}\right]t} = 0.14 A_0$$

$$\frac{\ln(\frac{1}{2})}{5730} t = \ln(0.14)$$

$$t = 5730 \frac{\ln(0.14)}{\ln(1/2)}$$

$$t \doteq 16253 \text{ yrs}$$