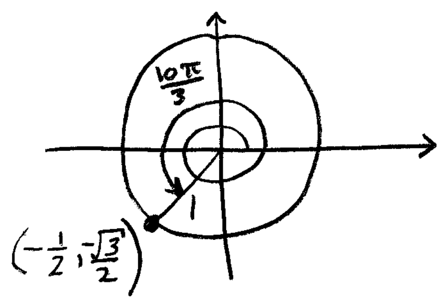
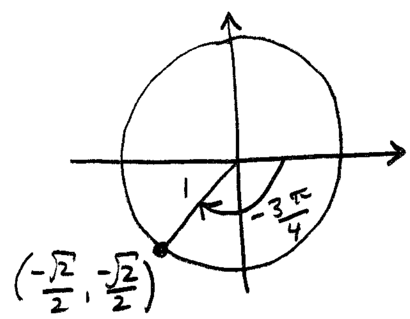
**Question 1:**

**(a)**[**2 points**] Convert $-405°$ to radians.

$$(-405°)\left(\frac{\pi \text{ radians}}{180°}\right) = \boxed{\frac{-9\pi}{4}}$$

**(b)**[**4 points**] Find the exact value of $\sec(10\pi/3)\sin(-3\pi/4)$.



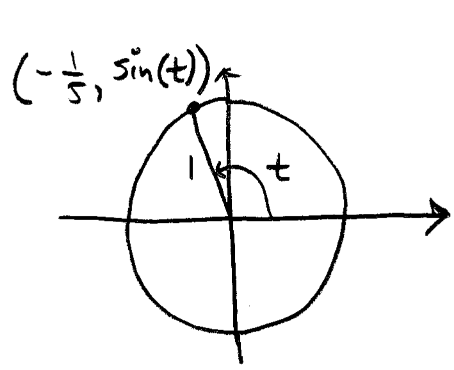

$$\therefore \sec\left(\frac{10\pi}{3}\right)\sin\left(-\frac{3\pi}{4}\right) = \frac{1}{\cos\left(\frac{10\pi}{3}\right)} \cdot \sin\left(\frac{-3\pi}{4}\right)$$

$$= \frac{1}{\left(-\frac{1}{2}\right)} \cdot \frac{-\sqrt{2}}{2}$$

$$= (-2)\left(\frac{-\sqrt{2}}{2}\right)$$

$$= \boxed{\sqrt{2}}$$

**(c)**[**4 points**] If $\cos(t) = -1/5$, find all possible values of $\sin(t)$.




$$\therefore \left(\frac{1}{5}\right)^2 + \sin^2(t) = 1$$

$$\sin(t) = \pm\sqrt{1 - \left(-\frac{1}{5}\right)^2}$$

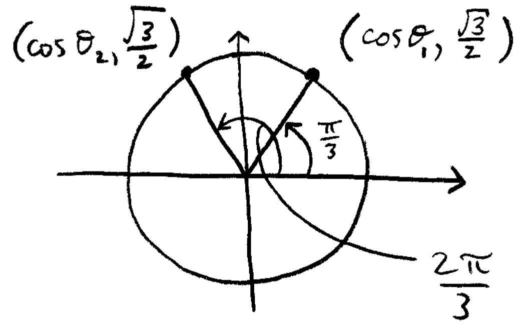
$$= \pm\sqrt{1 - \frac{1}{25}}$$

$$= \pm\sqrt{\frac{24}{25}}$$

$$= \boxed{\pm\frac{2\sqrt{6}}{5}}$$

**Question 2:**

**(a)[3 points]** Find all angles $0 \leq \theta < 2\pi$ such that $\sin\theta = \sqrt{3}/2$.




$$\boxed{\therefore \quad \theta = \frac{\pi}{3}, \frac{2\pi}{3}}$$

**(b)[3 points]** Find the exact value of $\sin(11\pi/12)$ (note: 2/3+1/4=11/12).

$$\sin\left(\frac{11\pi}{12}\right) = \sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$$

$$= \sin\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

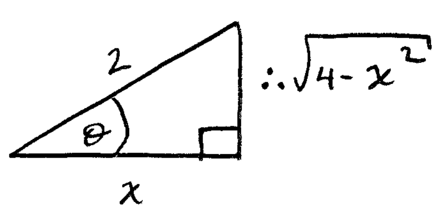
$$= \boxed{\frac{\sqrt{2}(\sqrt{3}-1)}{4}}$$

**(c)[4 points]** Simplify to an expression which does not contain trigonometric functions:

$$\sin(\arccos(x/2))$$

let $\theta = \arccos\left(\frac{x}{2}\right)$,

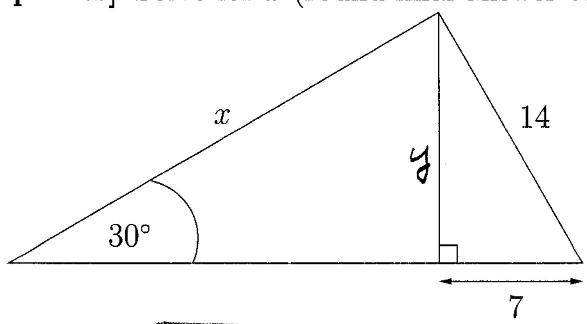
$\therefore \cos\theta = \frac{x}{2}$, where $0 \leq \theta \leq \pi$




$$\therefore \quad \sin\left(\arccos\left(\frac{x}{2}\right)\right) = \sin(\theta) = \boxed{\frac{\sqrt{4-x^2}}{2}}$$

**Question 3:**

**(a)**[**4 points**] Solve for $x$ (round final answer to one decimal):

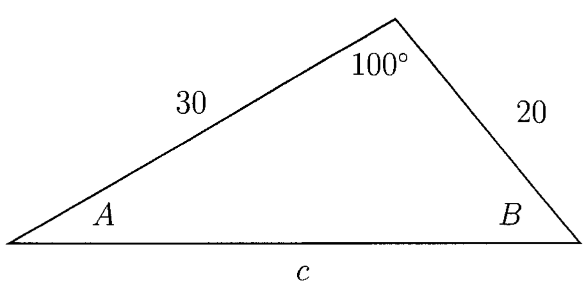



$y = \sqrt{14^2 - 7^2} = \sqrt{147} = 7\sqrt{3}$

$\sin(30°) = \frac{y}{x}$, $\therefore$ $x = \frac{y}{\sin(30°)} = \frac{7\sqrt{3}}{(\frac{1}{2})}$

$\boxed{= 14\sqrt{3} \doteq 24.2}$

**(b)**[**4 points**] Find all remaining sides and angles in the following figure (round final answers to one decimal):




$c^2 = a^2 + b^2 - 2ab\cos C$

$= (20)^2 + (30)^2 - 2(20)(30)\cos(100)$

$\therefore$ $c = \sqrt{1300 - 1200\cos(100°)} \doteq \boxed{38.8}$

$\frac{\sin A}{20} = \frac{\sin(100°)}{c}$, $\therefore$ $A = \sin^{-1}\left[\frac{20\sin(100°)}{\sqrt{1300 - 1200\cos(100°)}}\right]$

$\boxed{A \doteq 30.5°}$

$\frac{\sin B}{30} = \frac{\sin(100°)}{c}$, $\therefore$ $B = \sin^{-1}\left[\frac{30\sin(100°)}{\sqrt{1300 - 1200\cos(100°)}}\right] \doteq \boxed{49.5°}$

**(c)**[**2 points**] Find the exact value of $\log_{\frac{1}{2}} 16$

$\log_{\frac{1}{2}} 16 = \frac{\log_2 16}{\log_2 \frac{1}{2}} = \frac{4}{-1} = \boxed{-4}$

**Question 4:**

**(a)[2 points]** Find the $x$ intercept of the graph of $y = \log_7(2x-3) - 2$.

$0 = \log_7(2x-3) - 2$

$2 = \log_7(2x-3)$

$49 = 2x - 3$

$x = \frac{49+3}{2} = 26$ , $\boxed{\therefore (26,0)}$

**(b)[2 points]** Simplify:

$$\ln\left(\frac{1}{4}e^{3x}\right) - \ln\left(e^{2x}\right) + \ln 4$$

$= \ln(\frac{1}{4}) + \ln(e^{3x}) - \ln(e^{2x}) + \ln 4$

$= \underbrace{\cancel{\ln 1}}_{0} - \cancel{\ln 4} + 3x - 2x + \cancel{\ln 4}$

$= \boxed{x}$

**(c)[3 points]** Let $\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 3 & -5 \\ -9 & 2 \end{bmatrix}$, and $\mathbf{C} = \begin{bmatrix} -2 & 2 \\ 4 & -1 \end{bmatrix}$. Compute $(\mathbf{B} - 2\mathbf{C})\,\mathbf{A}^{\mathrm{T}}$.

$B - 2C = \begin{bmatrix} 7 & -9 \\ -17 & 4 \end{bmatrix}$

$\therefore \quad (B-2C)A^T = \begin{bmatrix} 7 & -9 \\ -17 & 4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix} = \boxed{\begin{bmatrix} 23 & -18 \\ -38 & 8 \end{bmatrix}}$

**(d)[3 points]** Let $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$. Find $\mathbf{A}^{-1}$.

$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right]$

$(-1)R_2: \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{array}\right]$

$(-2)R_2 + R_1: \left[\begin{array}{cc|cc} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -1 \end{array}\right]$

$\boxed{\therefore A^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}}$

**Question 5:**

**(a)**[**3 points**] Find the 11$^{\text{th}}$ term of the arithmetic sequence $\frac{7}{6}, \frac{5}{6}, \ldots$

$a_n = a + (n-1)d$

$a = \frac{7}{6}$,

$d = \frac{5}{6} - \frac{7}{6} = \frac{-2}{6} = \frac{-1}{3}$

$n = 11$

$\therefore\ a_{11} = \frac{7}{6} + 10\left(\frac{-1}{3}\right)$

$= \frac{7}{6} - \frac{20}{6}$

$= \boxed{\frac{-13}{6}}$

**(b)**[**3 points**] A geometric sequence has $a_3 = 1/2$ and $a_8 = -512$. What is $a_6$?

$a_3 = \frac{1}{2} = ar^2$

$a_8 = -512 = ar^7$

$\therefore\ \frac{ar^7}{ar^2} = \frac{-512}{\left(\frac{1}{2}\right)}$

$r^5 = -1024$

$\therefore\ r = -4$

$\therefore\ a_6 = a_3 \cdot r^3$

$= \left(\frac{1}{2}\right)(-4)^3$

$= \frac{-64}{2} = \boxed{-32}$

**(c)**[**4 points**] An arithmetic series has first term 7, last term $-47$ and common difference between terms of $d = -3$. Find the sum of the series.

$a_1 = 7$

$d = -3$

$a_n = a + (n-1)d = -47$

$7 + (n-1)(-3) = -47$

$n - 1 = \frac{-47-7}{-3}$

$\therefore\ n = 19$

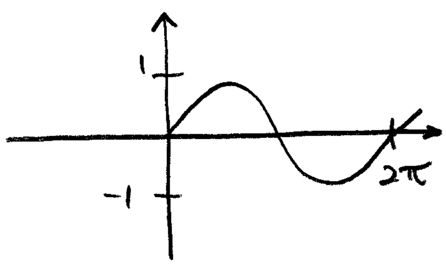
$\therefore\ S_{19} = 19\left(\frac{7 + (-47)}{2}\right) = \boxed{-380}$

**Question 6:**

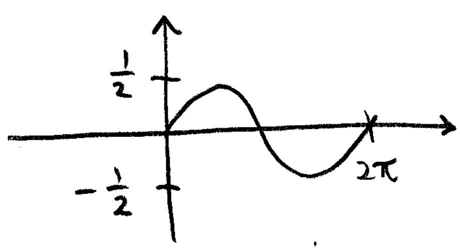
**(a)** [**7 points**] Carefully sketch the graph of $f(x) = \frac{1}{2}\sin\left(2x - \frac{\pi}{2}\right) - \frac{1}{2}$ showing at least one complete cycle of the function. Label and indicate the scale on your axes.

$y = \frac{1}{2}\sin\left[2\left(x - \frac{\pi}{4}\right)\right] - \frac{1}{2}$

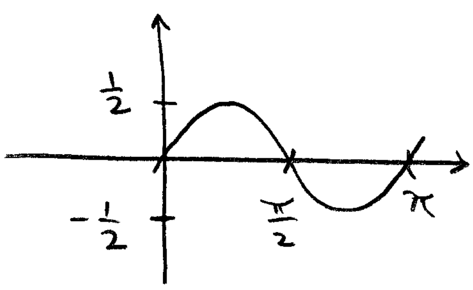
① $y = \sin(x)$ :



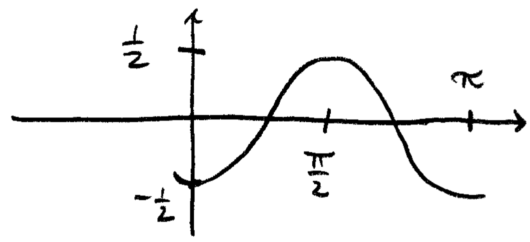

② $y = \frac{1}{2}\sin(x)$ :



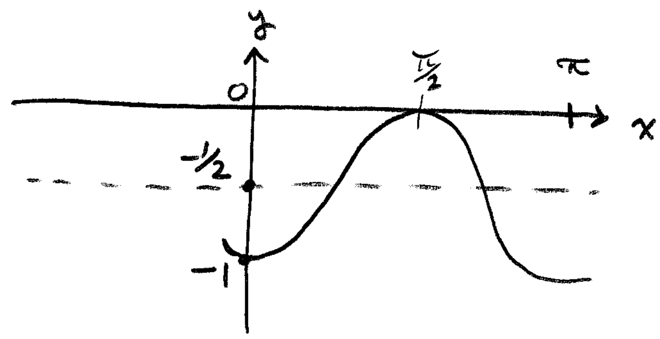

③ $y = \frac{1}{2}\sin[2x]$




④ $y = \frac{1}{2}\sin\left[2\left(x - \frac{\pi}{4}\right)\right]$




⑤ $y = \frac{1}{2}\sin\left[2\left(x - \frac{\pi}{4}\right)\right] - \frac{1}{2}$



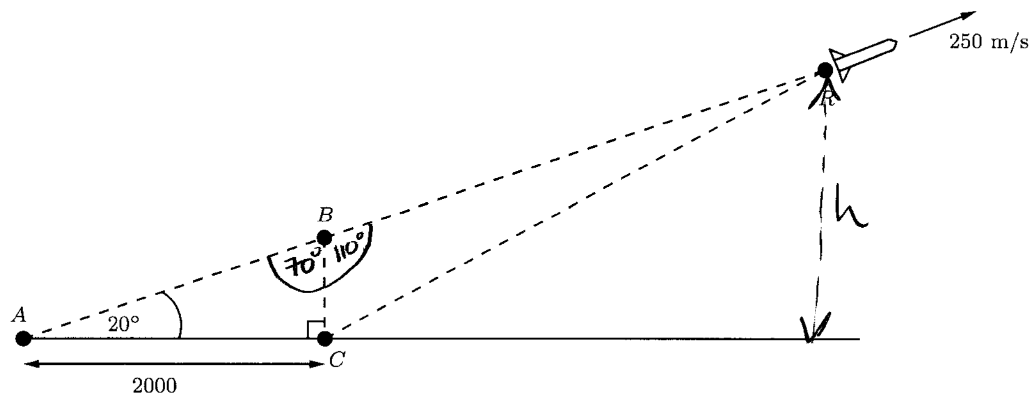

**(b)** [**3 points**] State the amplitude, period and phase-shift of the function graphed in (a).

Amplitude $= \left|\frac{1}{2}\right| = \boxed{\frac{1}{2}}$

Period $= \frac{2\pi}{2} = \boxed{\pi}$

Phase-shift $= \boxed{\frac{\pi}{4}}$

**Question 7:** A rocket traveling at 250 metres per second is climbing at an angle of 20° as shown in the figure below. A radar station at point $C$ located 2000 metres from the launch point $A$ is tracking the rocket.




**(a)** [**3 points**] What is the distance from the launch point $A$ to the rocket at $R$ three seconds after the rocket passes through point $B$? (round your answer to the nearest metre.)

$$BR = \left(250\ \frac{m}{s}\right)(3\,s) = 750\ m$$

$$AB = \frac{2000}{\cos(20^\circ)}$$

$$\therefore\ AR = 750 + \frac{2000}{\cos(20^\circ)} \doteq \boxed{2878\ m}$$

**(b)** [**4 points**] How far is the rocket at $R$ from the radar station $C$ at this same instant? (round your answer to the nearest metre.)

$$BC = 2000\tan(20^\circ)$$

$$(CR)^2 = (BC)^2 + (BR)^2 - 2(BC)(BR)\cos(110^\circ)$$

$$\therefore\ CR = \sqrt{(2000\tan(20))^2 + (750)^2 - 2(2000\tan(20))(750)\cos(110^\circ)}$$

$$\doteq \boxed{1211\ m}$$

**(c)** [**3 points**] How high above the ground is the rocket at this same instant? (round your answer to the nearest metre.)

$$h = AR\sin(20^\circ)$$

$$= \left[750 + \frac{2000}{\cos(20^\circ)}\right]\sin(20^\circ)$$

$$\doteq \boxed{984\ m}$$

**Question 8:** One population has size $P_1(t)$ at time $t$ years given by $P_1(t) = 1000e^{0.05t}$. A second population has size $P_2(t)$ at time $t$ years given by $P_2(t) = 800e^{0.08t}$.

**(a)** [**3 points**] What is the doubling time of the first population? (round your answer to one decimal.)

$$2000 = 1000\,e^{0.05t}$$

$$2 = e^{0.05t}$$

$$\ln 2 = 0.05t$$

$$t = \frac{\ln 2}{0.05} \doteq \boxed{13.9 \text{ years}}$$

**(b)** [**3 points**] How many years does it take the second population to reach 2500 in size? (round your answer to one decimal.)

$$2500 = 800\,e^{0.08t}$$

$$\frac{2500}{800} = e^{0.08t}$$

$$0.08\,t = \ln\left(\frac{25}{8}\right)$$

$$\therefore\ t = \frac{\ln\left(\frac{25}{8}\right)}{0.08} \doteq \boxed{14.2 \text{ years}}$$

**(c)** [**4 points**] At what time $t$ will both populations be equal in size? (round your answer to one decimal.)

$$1000\,e^{0.05t} = 800\,e^{0.08t}$$

$$\frac{1000}{800} = \frac{e^{0.08t}}{e^{0.05t}}$$

$$\frac{5}{4} = e^{0.03t}$$

$$\ln\left(\frac{5}{4}\right) = 0.03t$$

$$\therefore\ t = \frac{\ln\left(\frac{5}{4}\right)}{0.03} \doteq \boxed{7.4 \text{ yrs}}$$

**Question 9:**

**(a)**[**5 points**] Solve for $x$:

$$\log_{10}(3x) - \log_{10}(x+1) = \log_{10} x \ .$$

$$\log_{10}(3x) = \log_{10}(x) + \log_{10}(x+1)$$

$$\log_{10}(3x) = \log_{10}[x(x+1)]$$

$$3x = x^2 + x$$

$$\therefore \quad x^2 - 2x = 0 .$$

$$x(x-2) = 0$$

$$x = 0, \quad x = 2 ,$$

Check:

$x=0$: $\underbrace{\log_{10}(3\cdot 0)}_{\text{not defined!}} - \log_{10}(0+1)$ $\Big\{$ $\log_{10}(0)$ $\Big\}$ $\therefore x = 0$ is not a solution.

$x=2$:

$$\left.\begin{array}{lcl} \log_{10}(3\cdot 2) - \log_{10}(2+1) & \Big\{ & \log_{10}(2) \\ \log_{10}(6) - \log_{10}(3) & & \log_{10}(2) \\ \log_{10}\left(\frac{6}{3}\right) & & \log_{10}(2) \\ \log_{10}(2) & \stackrel{\checkmark}{=} & \log_{10}(2) \end{array}\right\}$$

$\boxed{\therefore x = 2 \text{ is the only solution.}}$

**(b)**[**5 points**] Find all solutions $0 \le t < 2\pi$ to

$$2\sin^2(t) + \sin(t) - 1 = 0 \ .$$

Let $w = \sin(t)$ : $2w^2 + w - 1 = 0$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-1 \pm 3}{4}$$

$$= \frac{1}{2}, \quad -1$$

$$\therefore \quad \sin(t) = \frac{1}{2} , \qquad \sin t = -1$$

$$\boxed{\therefore t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2} .}$$

**Question 10 [10 points]:** Solve the following system of equations **using matrix reduction** (no credit will be given for using any other method):

$$
\begin{aligned}
5x - 10y + 5z &= -15\\
-5x + 8y - 7z &= -5\\
10x - 18y + 13z &= -3
\end{aligned}
$$

$$\left[\begin{array}{ccc|c} 5 & -10 & 5 & -15 \\ -5 & 8 & -7 & -5 \\ 10 & -18 & 13 & -3 \end{array}\right]$$

$\frac{1}{5}R_1$: $$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ -5 & 8 & -7 & -5 \\ 10 & -18 & 13 & -3 \end{array}\right]$$

$5R_1+R_2$:
$-10R_1+R_3$:

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 0 & -2 & -2 & -20 \\ 0 & 2 & 3 & 27 \end{array}\right]$$

$-\frac{1}{2}R_2$:

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 0 & 1 & 1 & 10 \\ 0 & 2 & 3 & 27 \end{array}\right]$$

$2R_2+R_1$:
$-2R_2+R_3$:

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 17 \\ 0 & 1 & 1 & 10 \\ 0 & 0 & 1 & 7 \end{array}\right]$$

$-3R_3+R_1$:
$-R_3+R_2$:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 7 \end{array}\right]$$

$$\boxed{\begin{aligned} \therefore\ x &= -4\\ y &= 3\\ z &= 7 \end{aligned}}$$

**You may find some of the following formulas useful:**

$$\sin^2(A) + \cos^2(A) = 1$$

$$\tan^2(A) + 1 = \sec^2(A)$$

$$1 + \cot^2(A) = \csc^2(A)$$

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

$$\sin(2A) = 2\sin(A)\cos(A)$$

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

$$\cos(2A) = 1 - 2\sin^2(A)$$

$$\cos(2A) = 2\cos^2(A) - 1$$

$$\sin^2(A/2) = \frac{1-\cos(A)}{2} \qquad \cos^2(A/2) = \frac{1+\cos(A)}{2}$$

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a_n = a + (n-1)d \qquad a_n = ar^{n-1}$$

$$S_n = n\,\frac{(a_1 + a_n)}{2} \qquad S_n = \frac{n[2a + (n-1)d]}{2} \qquad S_n = \frac{a(1-r^n)}{1-r}$$