Question 1 [10 points]:
Find the quotient $q(x)$ and remainder $r(x)$ when the polynomial $f(x) = 14x^4 - 12x^2 + 6$ is divided by $d(x) = x^2 - 1$. Express your answer in the form $f(x) = d(x)q(x) + r(x)$.

$$
\begin{array}{r}
14x^2 + 2 \\
x^2 + 0x - 1 \overline{\big)\, 14x^4 + 0x^3 - 12x^2 + 0x + 6} \\
-(14x^4 + 0x^3 - 14x^2) \\
\hline
2x^2 + 0x + 6 \\
-(2x^2 + 0x - 2) \\
\hline
8
\end{array}
$$

$$\therefore\ 14x^4 - 12x^2 + 6 = (x^2-1)(14x^2+2) + 8$$

Question 2:

(a)[5 points] Find the remainder upon dividing $5x^3 + x^2 - 4x - 6$ by $x + 1$.

$$\begin{array}{r|rrrr} -1 & 5 & 1 & -4 & -6 \\ & & -5 & 4 & 0 \\ \hline & 5 & -4 & 0 & \boxed{-6} \end{array}$$

$\boxed{\therefore \text{ remainder is } -6.}$

(b)[5 points] Let $f(x) = x^7 - 3x^5 + 2x^3 - x + 10$. Find $f(5)$. (It is easiest to do this by synthetic division.)

$$\begin{array}{r|rrrrrrrr} 5 & 1 & 0 & -3 & 0 & 2 & 0 & -1 & 10 \\ & & 5 & 25 & 110 & 550 & 2760 & 13800 & 68995 \\ \hline & 1 & 5 & 22 & 110 & 552 & 2760 & 13799 & \boxed{69005} \end{array}$$

$\boxed{\therefore f(5) = 69\,005}$

Question 3 [10 points]: Factor completely:

$$f(x) = x^4 - 6x^3 + 4x^2 + 6x - 5$$

$\frac{p}{s} = 1, -1, 5, -1$

$$\begin{array}{r|rrrrr} 1 & 1 & -6 & 4 & 6 & -5 \\ & & 1 & -5 & -1 & 5 \\ \hline & 1 & -5 & -1 & 5 & \boxed{0} \end{array}$$

$\leftarrow \therefore$ $x-1$ is a factor

$\therefore \; f(x) = (x-1)\underbrace{(x^3 - 5x^2 - x + 5)}_{}$

$\frac{p}{s} = 1, -1, 5, -5$

$$\begin{array}{r|rrrr} 1 & 1 & -5 & -1 & 5 \\ & & 1 & -4 & -5 \\ \hline & 1 & -4 & -5 & \boxed{0} \end{array}$$

$\leftarrow \therefore$ $(x-1)$ is a factor

$\therefore \; f(x) = (x-1)(x-1)(x^2 - 4x - 5)$

$$\boxed{f(x) = (x-1)(x-1)(x+1)(x-5)}$$

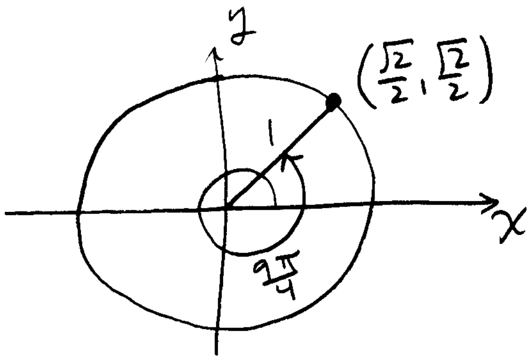
Question 4:

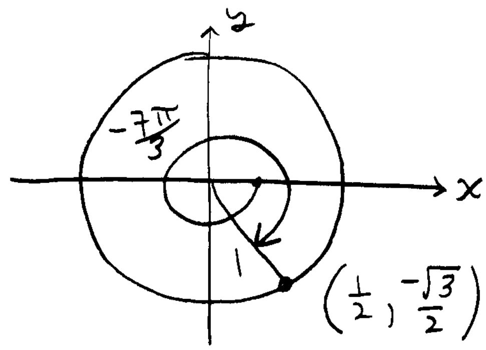
(a)[3 points] Convert 875° to radians.

$$875° = (875°)\left(\frac{\pi \text{ radians}}{180°}\right)$$
$$= \boxed{\frac{175\pi}{36} \text{ radians}}$$

(b)[3 points] Find the exact value of

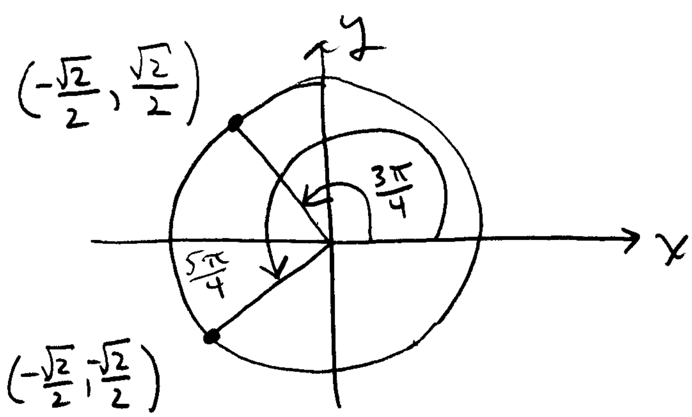
$$\sin\left(\frac{9\pi}{4}\right)\cos\left(\frac{-7\pi}{3}\right)$$





$$\therefore \sin\left(\frac{9\pi}{4}\right)\cos\left(-\frac{7\pi}{3}\right) = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \boxed{\frac{\sqrt{2}}{4}}$$

(c)[4 points] Find all angles $0 \leq t < 2\pi$ such that $\cos(t) = -\sqrt{2}/2$

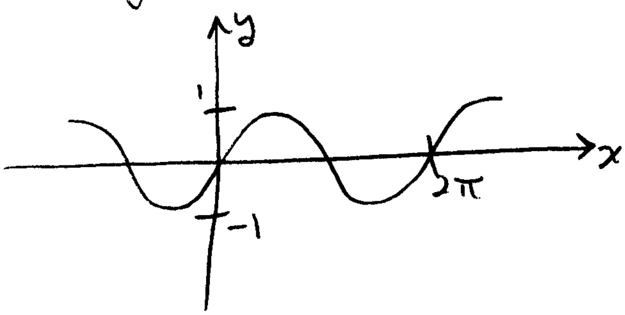


$$\boxed{\therefore\ t = \frac{3\pi}{4}, \frac{5\pi}{4}}$$

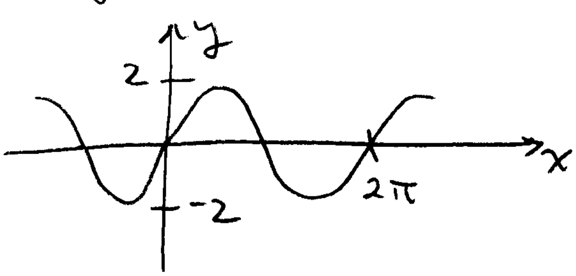
Question 5:

(a)[7 points] Neatly sketch the graph of $y = 2\sin(2x) - 3$. Show the scale on the x and y axes.

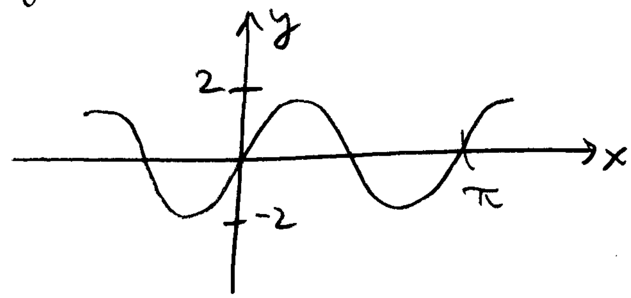
① $y = \sin(x)$:



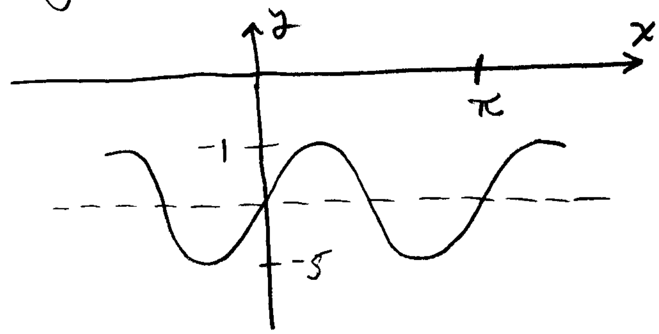
② $y = 2\sin(x)$:



③ $y = 2\sin(2x)$



④ $y = 2\sin(2x) - 3$



(b)[3 points] State the amplitude, period and phase shift of the function from part (a).

amplitude $= 2$

period $= \frac{2\pi}{2} = \pi$

phase shift $= 0$.