Question 1: For the function

$$f(x) = \frac{x^2 - 1}{x^2 - 4}$$

(a)[3 points] State the domain.

$f(x)$ is not defined if $x^2-4=0$, i.e. $x = 2, -2$.

$\therefore$ domain is all real x except $x = 2, -2$,

i.e. $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

(b)[2 points] Find the y-intercept of the graph of f.

$$f(0) = \frac{0^2-1}{0^2-4} = \frac{1}{4}$$

$\therefore$ y-intercept is $(0, \frac{1}{4})$

(c)[2 points] Find the x-intercepts of the graph of f.

$f(x) = 0$ if $x^2-1=0$

$\therefore$ $x = 1, -1$

$\therefore$ x-intercepts are $(1,0), (-1,0)$

(d)[3 points] Determine if f is even.

$$f(x) - f(-x) = \frac{x^2-1}{x^2-4} - \frac{(-x)^2-1}{(-x)^2-4} = \frac{x^2-1}{x^2-4} - \frac{x^2-1}{x^2-4} = 0.$$

$\therefore$ f is even.

Question 2:

(a)[**5 points**] Find the point of intersection of the lines

$$2x - y + 13 = 0 \quad (1)$$
$$x + 2y - 1 = 0 \quad (2)$$

Using (2), $x = 1 - 2y$.

Substituting into (1): $2(1-2y) - y + 13 = 0$

$$2 - 4y - y + 13 = 0$$
$$-5y = -15$$
$$y = 3$$
$$\therefore \quad x = 1 - 2(3) = -5$$

$\therefore$ Point of intersection is $(-5, 3)$

(b)[**5 points**] Find the equation of the line through $(-3, 1)$ which is perpendicular to the line $x + 2y - 2 = 0$.

$$x + 2y - 2 = 0$$
$$2y = -x + 2$$
$$y = -\tfrac{1}{2}x + 1$$

slope $-\frac{1}{2}$

$\therefore$ slope of line we seek is $m = \dfrac{-1}{(-\frac{1}{2})} = 2$

$\therefore$ Equation of line of slope 2 through $(-3, 1)$ is

$$y - 1 = 2(x + 3)$$

or $\quad y = 2x + 7$

Question 3: For the quadratic function

$$f(x) = -x^2 + 6x - 5$$

(a)[3 points] Put the function in standard form.

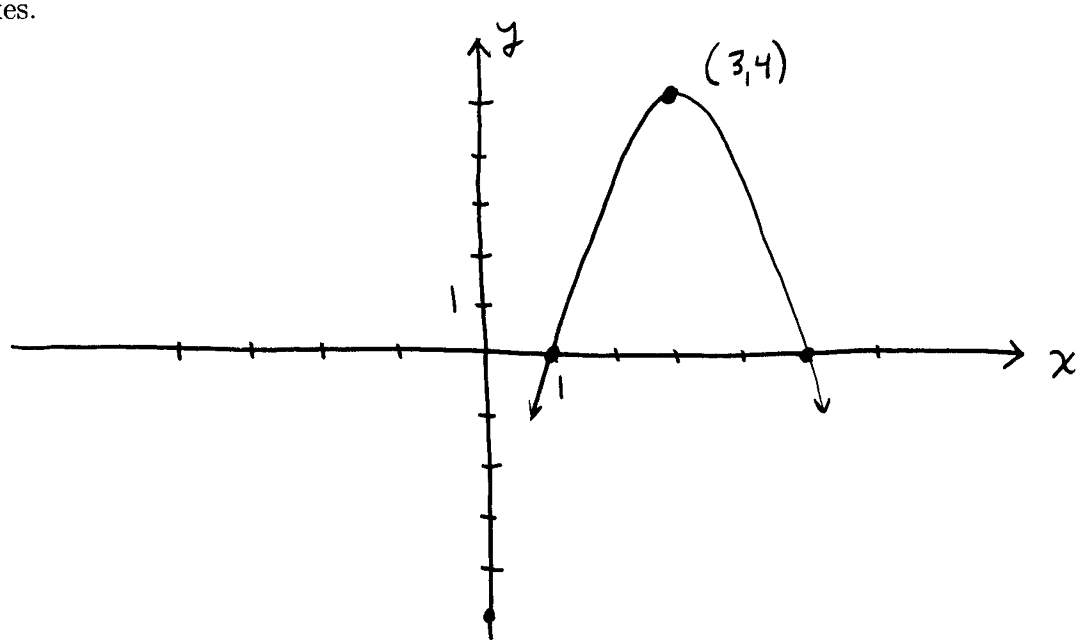
$$f(x) = -[x^2 - 6x + 5]$$
$$= -[(x-3)^2 - 9 + 5]$$
$$= -(x-3)^2 + 4$$

(b)[3 points] State the vertex and axis of symmetry.

Vertex : $(3, 4)$

axis of symmetry : $x = 3$

(c)[4 points] Sketch the graph of f. Label the vertex, and indicate the scale on the x and y axes.



Question 4:

(a)[4 points] Find and simplify $(f \circ g)(x)$ where $f(x) = \dfrac{x+2}{x}$ and $g(x) = \dfrac{2}{x}$.

$$(f \circ g)(x) = f(g(x)) = \frac{\left(\frac{2}{x}\right) + 2}{\left(\frac{2}{x}\right)} \quad \ldots (*)$$

$$= \frac{\left(\frac{2+2x}{x}\right)}{\frac{2}{x}}$$

$$= \frac{2+2x}{x} \cdot \frac{x}{2} = 1 + x$$

(b)[3 points] State the domain of $f \circ g$ from part (a).

Using $(*)$ in (a), domain of $f \circ g$ is all real x except $x = 0$, i.e. $(-\infty, 0) \cup (0, \infty)$.

(c)[3 points] Suppose $F(x) = \dfrac{\sqrt{x^2+1}+2}{x^2}$. Find functions f and g such that $F = f \circ g$. (There are many possible answers here).

$g(x) = x^2$ OR $g(x) = x^2 + 1$

$f(x) = \dfrac{\sqrt{x+1}+2}{x}$ $\qquad$ $f(x) = \dfrac{\sqrt{x}+2}{x-1}$

Question 5:

(a)[**5 points**] The function $f(x) = \dfrac{4x}{x+1}$ has domain $(-\infty, -1) \cup (-1, \infty)$ and range $(-\infty, 4) \cup (4, \infty)$. Find f^{-1} and state its domain and range.

$$y = \frac{4x}{x+1}$$

$$x = \frac{4y}{y+1}$$

$$xy + x = 4y$$

$$4y - xy = x$$

$$y(4-x) = x$$

$$y = \frac{x}{4-x}$$

$\therefore\ f^{-1}(x) = \dfrac{x}{4-x}$ } Domain of f^{-1} = range of f = $(-\infty,4)\cup(4,\infty)$
Range of f^{-1} = domain of f = $(-\infty,-1)\cup(1,\infty)$

(b)[**5 points**] Below is the graph $y = f(x)$ for some function f. Sketch the graph of $y = f^{-1}(x)$ on the same coordinate axes. Your graph must be accurate to receive full marks.