(1)[7 points] Factor completely:

$$f(x) = 8x^3 + 5x^2 - 11x + 3$$

Given that $x - \frac{3}{8}$ is a factor, $f(\frac{3}{8}) = 0$, so

$\frac{3}{8}$ | 8 5 -11 3

3 3 -3

8 8 -8 0

$\therefore f(x) = (x - \frac{3}{8})(8x^2 + 8x - 8)$

$= (8x - 3)(x^2 + x - 1)$

Now solve $x^2 + x - 1 = 0$: $x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$

$= -1 \pm \frac{\sqrt{5}}{2}$

$\therefore$ $f(x)$ factors as

$$f(x) = (8x-3)\left(x - \frac{-1+\sqrt{5}}{2}\right)\left(x - \frac{-1-\sqrt{5}}{2}\right)$$

(2)[8 points] Solve:

$$f(x) = 2x^4 + 7x^3 - 8x^2 - 25x - 6 = 0$$

Given that $x=2$ is one solution, $x-2$ is a factor of $f(x)$:

$$\begin{array}{r|cccc} 2 & 2 & 7 & -8 & -25 & -6 \\ & & 4 & 22 & 28 & 6 \\ \hline & 2 & 11 & 14 & 3 & \boxed{0} \end{array}$$

$\therefore f(x) = (x-2)(2x^3+11x^2+14x+3)$

Given that $x = -\frac{3}{2}$ is a solution, $x+\frac{3}{2}$ is a factor of $2x^3+11x^2+14x+3$:

$$\begin{array}{r|cccc} -\frac{3}{2} & 2 & 11 & 14 & 3 \\ & & -3 & -12 & -3 \\ \hline & 2 & 8 & 2 & \boxed{0} \end{array}$$

$\therefore f(x) = (x-2)\left(x+\frac{3}{2}\right)(2x^2+8x+2)$

$= (x-2)(2x+3)(x^2+4x+1)$

Remaining solutions are given by

$x^2+4x+1=0$

$$x = \frac{-4 \pm \sqrt{4^2-4(1)(1)}}{2} = \frac{-4 \pm 2\sqrt{3}}{2} = -2 \pm \sqrt{3}$$

$\therefore$ Solutions are $x = 2, -\frac{3}{2}, -2+\sqrt{3}, -2-\sqrt{3}$.