Question 1:

(a)[**4 points**] Expand and simplify:

$$3(x+2)^2 - 2(x+3)(x-5)$$

$$= 3(x^2+4x+4) - 2(x^2-2x-15)$$

$$= 3x^2+12x+12-2x^2+4x+30$$

$$= \boxed{x^2+16x+42}$$

(b)[**3 points**] Factor completely:

$$6x^2+5x-6$$

$(6)(-6) = -36$

$9-4 = 5$

$$= 6x^2+9x-4x-6$$

$$= 3x(2x+3) - 2(2x+3)$$

$$= \boxed{(3x-2)(2x+3)}$$

(c)[**3 points**] Write as a simplified fraction using only positive exponents:

$$(y^{-1}-x^{-1})(x-y)^{-1}$$

$$= \left(\frac{1}{y}-\frac{1}{x}\right)\left(\frac{1}{x-y}\right)$$

$$= \left(\frac{x-y}{xy}\right)\left(\frac{1}{x-y}\right)$$

$$= \boxed{\frac{1}{xy}}$$

Question 2:

(a)[**4 points**] Solve for x:

$$2x^2 - x - 5 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-5)}}{2(2)}$$

$$= \frac{1 \pm \sqrt{41}}{4}$$

$$= \boxed{\frac{1+\sqrt{41}}{4}, \frac{1-\sqrt{41}}{4}}$$

(b)[**3 points**] Rationalize the denominator:

$$\frac{1}{\sqrt{x+h} - \sqrt{x}} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

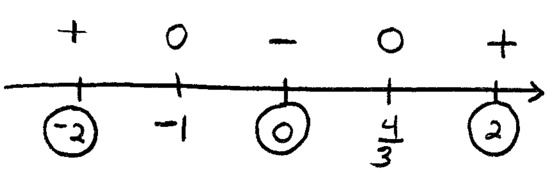
$$= \frac{\sqrt{x+h} + \sqrt{x}}{x + h - x}$$

$$= \boxed{\frac{\sqrt{x+h} + \sqrt{x}}{h}}$$

(c)[**3 points**] Solve and state your answer using interval notation:

$$\left(x - \frac{4}{3}\right)(x+1) \leq 0$$

$\left(x - \frac{4}{3}\right)(x+1)$:



$\therefore \left(x - \frac{4}{3}\right)(x+1) \leq 0$ on $\left[-1, \frac{4}{3}\right]$

Question 3:

(a)[4 points] Solve and state your answer using interval notation

$$|3x - 8| \leq 7$$

$$-7 \leq 3x - 8 \leq 7$$

$$1 \leq 3x \leq 15$$

$$\frac{1}{3} \leq x \leq \frac{15}{3}$$

$$\boxed{\frac{1}{3} \leq x \leq 5}$$

(b)[3 points] Find the equation of the line through $(-1, 4/3)$ and $(3, 10/3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10/3 - 4/3}{3 - (-1)} = \frac{2}{4} = \frac{1}{2}$$

$$\therefore \boxed{y - \frac{10}{3} = \frac{1}{2}(x - 3)}$$

or $\boxed{y = \frac{1}{2}x + \frac{11}{6}}$

(c)[3 points] Find the distance between the points $(5, 2)$ and $(8, -2)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(5-8)^2 + (2-(-2))^2}$$

$$= \sqrt{9 + 16}$$

$$= \boxed{5}$$

Question 4:

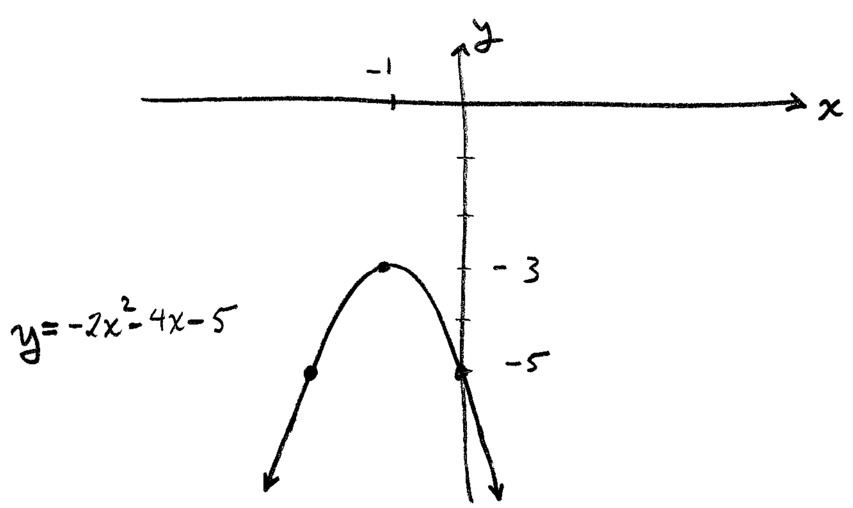
(a)[5 points] Put the quadratic function $f(x) = -2x^2 - 4x - 5$ into standard form by completing the square. State the vertex and axis of symmetry of the graph of $f(x)$.

$$f(x) = -2x^2 - 4x - 5$$
$$= -2\left[x^2 + 2x\right] - 5$$
$$= -2\left[(x+1)^2 - 1\right] - 5$$
$$= -2(x+1)^2 - 3$$

vertex $(-1, -3)$

axis of symmetry: $x = -1$

(b)[5 points] Use your result in (a) to sketch the graph of $y = -2x^2 - 4x - 5$. Label and indicate the scale on your axes, and label the vertex.



Question 5:

(a)[**5 points**] Find the point of intersection of the pair of lines

$$\left.\begin{array}{ll} ① & 2x - y = 2 \\ ② & x + y = -1/2 \end{array}\right\}$$

Using ②, $y = -\frac{1}{2} - x$; substituting into ①:

$$2x - \left(-\tfrac{1}{2} - x\right) = 2$$

$$2x + \tfrac{1}{2} + x = 2$$

$$3x = \tfrac{3}{2}$$

$$\therefore \quad x = \tfrac{1}{2}$$

$$\therefore \quad y = -\tfrac{1}{2} - \tfrac{1}{2} = -1$$

$$\boxed{\therefore \left(\tfrac{1}{2}, -1\right)}$$

(b)[**5 points**] Find the equation of the line through $(-2, 7)$ which is perpendicular to the line $y = \frac{1}{3}x - \frac{5}{7}$.

$m = \frac{1}{3}$, $\therefore$ slope of line we seek is $\frac{-1}{m} = -3$

$\therefore$ equation of line is $\boxed{y - 7 = -3(x+2)}$

or $\boxed{y = -3x + 1}$

Question 6:

(a)[3 points] Find the domain of $f(x) = \dfrac{\sqrt{x}}{\sqrt{4-x}}$.

Must have $x \geq 0$ & $4-x > 0$

$\therefore$ $x \geq 0$, $x < 4$

$\therefore$ $0 \leq x < 4$,

i.e. domain is $\boxed{[0,4)}$

(b)[4 points] Find the inverse $f^{-1}(x)$ of $f(x) = \dfrac{\sqrt{x}}{\sqrt{4-x}}$.

$$y = \frac{\sqrt{x}}{\sqrt{4-x}}$$

$x \leftrightarrow y$: $x = \dfrac{\sqrt{y}}{\sqrt{4-y}}$

$$x^2 = \frac{y}{4-y}$$

$$4x^2 - x^2 y = y$$

$$4x^2 = y + x^2 y$$

$$4x^2 = y(1+x^2)$$

$$y = \frac{4x^2}{1+x^2}$$

$$\boxed{\therefore \ f^{-1}(x) = \frac{4x^2}{1+x^2}}$$

(c)[3 points] If $f(x) = \dfrac{\sqrt{x}}{\sqrt{4-x}}$ and $g(x) = \dfrac{1}{x^2}$, find and simplify $(g \circ f)(x)$.

$$(g \circ f)(x) = g(f(x)) = \frac{1}{\left(\frac{\sqrt{x}}{\sqrt{4-x}}\right)^2} = \frac{4-x}{x} = \boxed{\frac{4}{x} - 1}$$

Question 7:

(a)[3 points] Determine if $x = 1$ is a zero of $x^4 + 2x^3 - 7x^2 - 8x + 12$.

$$\begin{array}{r|rrrrr} 1 & 1 & 2 & -7 & -8 & 12 \\ & & 1 & 3 & -4 & -12 \\ \hline & 1 & 3 & -4 & -12 & \boxed{0} \end{array} \leftarrow \text{remainder.}$$

$\therefore$ $x-1$ is a factor & $x=1$ is a zero

(b)[3 points] Find the remainder upon dividing $x^4 + 2x^3 - 7x^2 - 8x + 12$ by $x + 3$.

$$\begin{array}{r|rrrrr} -3 & 1 & 2 & -7 & -8 & 12 \\ & & -3 & 3 & 12 & -12 \\ \hline & 1 & -1 & -4 & 4 & \boxed{0} \end{array} \leftarrow \text{remainder}$$

$\therefore$ remainder is 0.

(c)[4 points] Use (a) and (b) to completely factor $x^4 + 2x^3 - 7x^2 - 8x + 12$.

Let $f(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$

using (a), $f(x) = (x-1)(x^3 + 3x^2 - 4x - 12)$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -4 & -12 \\ & & -3 & 0 & 12 \\ \hline & 1 & 0 & -4 & \boxed{0} \end{array}$$

$\therefore$ $f(x) = (x-1)(x+3)(x^2-4)$

$= \boxed{(x-1)(x+3)(x-2)(x+2)}$

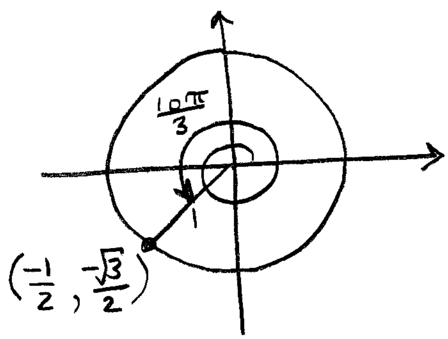
Question 8:

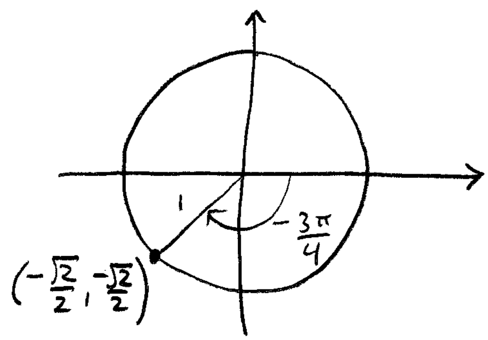
(a)[**3 points**] Convert $-405°$ to radians.

$$(-405°)\left(\frac{\pi \text{ radians}}{180°}\right) = -\frac{405}{180}\pi$$

$$= \boxed{-\frac{9\pi}{4} \text{ radians}}$$

(b)[**4 points**] Find $\sec(10\pi/3)\sin(-3\pi/4)$.





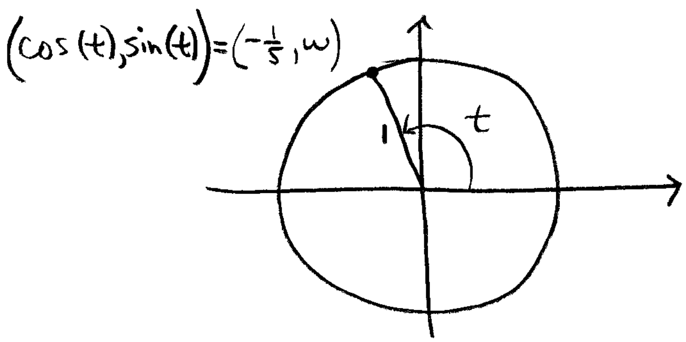
$$\sec\left(\frac{10\pi}{3}\right)\sin\left(-\frac{3\pi}{4}\right)$$

$$= \frac{1}{\cos\left(\frac{10\pi}{3}\right)} \cdot \sin\left(-\frac{3\pi}{4}\right)$$

$$= \frac{1}{\left(-\frac{1}{2}\right)} \cdot \left(-\frac{\sqrt{2}}{2}\right)$$

$$= \boxed{\sqrt{2}}$$

(c)[**3 points**] If $\cos(t) = -1/5$, find all possible values of $\sin(t)$.



$$\therefore \boxed{\sin(t) = \frac{2\sqrt{6}}{5}, \frac{-2\sqrt{6}}{5}}$$

$$\left(-\frac{1}{5}\right)^2 + w^2 = 1$$

$$\therefore w = \pm\sqrt{1 - \left(-\frac{1}{5}\right)^2}$$

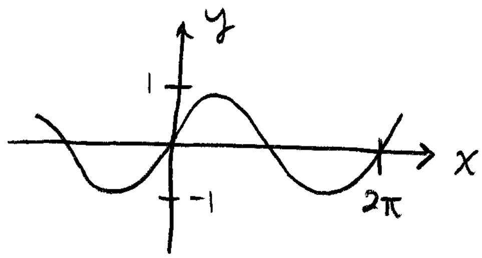
$$= \pm\sqrt{1 - \frac{1}{25}}$$

$$= \pm\frac{2\sqrt{6}}{5}$$

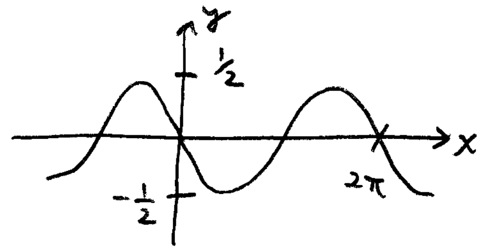
Question 9:

(a)[**7 points**] Carefully sketch the graph of $f(x) = -\frac{1}{2}\sin[2(x - \pi/4)]$. Label and indicate the scale on your axes.

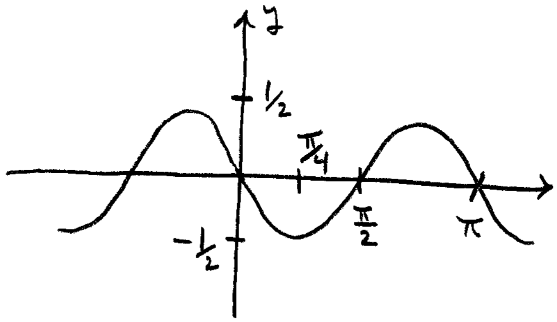
① $y = \sin(x)$:



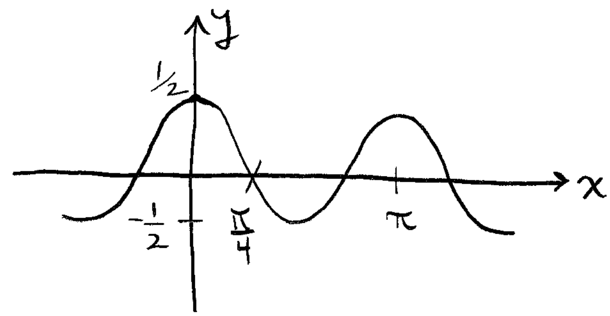
② $y = -\frac{1}{2}\sin(x)$:



③ $y = -\frac{1}{2}\sin(2x)$:



④ $y = -\frac{1}{2}\sin\left(2\left(x - \frac{\pi}{4}\right)\right)$



(b)[**3 points**] State the period, amplitude and phase-shift of the function graphed in (a).

amplitude $= \left|-\frac{1}{2}\right| = \frac{1}{2}$

period $= \frac{2\pi}{2} = \pi$

phase-shift $= \frac{\pi}{4}$

Question 10:

(a)[**4 points**] One population has size $P_1(t)$ at time t years given by $P_1(t) = 1000e^{0.05t}$. A second population has size $P_2(t)$ at time t years given by $P_1(t) = 800e^{0.08t}$. At what time t will the two populations be equal in size?

Solve $P_1(t) = P_2(t)$

$$1000e^{0.05t} = 800e^{0.08t}$$

$$\therefore \frac{1000\,e^{0.05t}}{800\,e^{0.08t}} = 1$$

$$\frac{10}{8}\,e^{-0.03t} = 1$$

$$e^{-0.03t} = \frac{8}{10}$$

$$\therefore \ln\left(e^{-0.03t}\right) = \ln\left(\frac{8}{10}\right)$$

$$-0.03t = \ln\left(\frac{4}{5}\right)$$

$$t = \frac{\ln(4/5)}{-0.03}$$

$$\boxed{\therefore t \doteq 7.4 \text{ yrs.}}$$

(b)[**6 points**] Solve for x:

$$\log_{10}(3x) - \log_{10}(x+1) = \log_{10} x$$

$$\log_{10}\left(\frac{3x}{x+1}\right) = \log_{10} x$$

$$\frac{3x}{x+1} = x$$

$$3x = x^2 + x$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$x = 0, 2$$

Check: $x=0$: $\log_{10}(3\cdot 0) - \log_{10}(0+1)$ | $\log_{10}(0)$; $\therefore x=0$ is not a solution.

↑ not defined!

$x=2$: $\log_{10}(3\cdot 2) - \log_{10}(2+1)$ | $\log_{10}(2)$

$\log_{10}\left(\frac{6}{3}\right)$ | $\log_{10}(2)$

$\log_{10} 2 = \log_{10}(2)$ ✓

$\therefore x=2$ is the only solution.