Math 122 Sec S08N03/04 - Quiz 4 Name: SOLUTIONS
Feb 18 2008 Stu#:

(1)[**6 points**] Evaluate

$$I=\int \tan^5 x\,dx$$

$$I = \int \frac{\sin^4 x}{\cos^5 x} \sin x\, dx$$

$$= -\int \frac{(1-\cos^2 x)^2(-\sin x)}{\cos^5 x}\, dx$$

let $u = \cos x$

$du = -\sin x\, dx$

$$\therefore I = -\int \frac{(1-u^2)^2}{u^5}\, du$$

$$= -\int \frac{1-2u^2+u^4}{u^5}\, du$$

$$= -\int u^{-5} - 2u^{-3} + u^{-1}\, du$$

$$= -\left[\frac{u^{-4}}{-4} - 2\frac{u^{-2}}{-2} + \ln|u|\right] + C$$

$$= \frac{1}{4}\frac{1}{\cos^4 x} - \frac{1}{\cos^2 x} - \ln|\cos x| + C$$

or

$$= \frac{1}{4}\sec^4 x - \sec^2 x + \ln|\sec x| + C$$

(2)[**3 points**] Evaluate

$$I = \left(\frac{1}{2}\right)\int \frac{(2)x}{\sqrt{x^2-7}}\, dx$$

let $u = x^2 - 7$

$du = 2x\, dx$

$$\therefore I = \frac{1}{2}\int u^{-1/2}\, du$$

$$= \cancel{\left(\frac{1}{2}\right)} \frac{u^{1/2}}{\cancel{\left(\frac{1}{2}\right)}} + C$$

$$= \sqrt{x^2-7} + C$$

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(3)[6 points] Evaluate

$$\int_{\sqrt{2}}^{2} \frac{1}{t^2\sqrt{t^2-1}}\,dt$$

$$I = \int \frac{1}{t^2\sqrt{t^2-1}}\,dt$$

let $t = 1 \cdot \sec\theta$

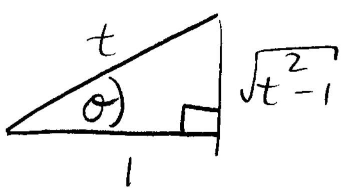
$dt = \sec\theta\tan\theta\,d\theta$

$$\therefore I = \int \frac{1 \cdot \sec\theta\tan\theta}{\sec^2\theta\sqrt{\sec^2\theta-1}}\,d\theta$$

$$= \int \frac{\sec\theta\cancel{\tan\theta}}{\sec^2\theta\,\cancel{\tan\theta}}\,d\theta$$

$$= \int \cos\theta\,d\theta$$

$$= \sin\theta$$



$$\therefore I = \frac{\sqrt{t^2-1}}{t}$$

$$\therefore \int_{\sqrt{2}}^{2} \frac{1}{t^2\sqrt{t^2-1}}\,dt$$

$$= \left[\frac{\sqrt{t^2-1}}{t}\right]_{\sqrt{2}}^{2}$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}-\sqrt{2}}{2}$$