Math 122 Sec S08N04 - Quiz 3 Name: SOLUTIONS.
Feb 11 2008 Stu#:

(1)[5 points] Use the definition of the definite integral in the form $\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$ to evaluate

$$\int_0^3 (3+x^2)\,dx\ .$$

Recall that

$$\sum_{i=1}^{n} c = cn \quad \text{where } c \text{ is a constant}, \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}\ .$$

$f(x) = 3 + x^2$

$\Delta x = \frac{3-0}{n} = \frac{3}{n}$

$x_i = a + i\Delta x = 0 + i\left(\frac{3}{n}\right) = \left(\frac{3}{n}\right) i$

$$\therefore \int_0^3 (3+x^2)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} (3 + x_i^2)\,\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(3 + \left(\frac{3}{n} i\right)^2\right)\left(\frac{3}{n}\right)$$

$$= \lim_{n\to\infty} \left[\left(\sum_{i=1}^{n} \frac{9}{n}\right) + \left(\sum_{i=1}^{n} \frac{27}{n^3} i^2\right)\right]$$

$$= \lim_{n\to\infty} \left[\left(\frac{9}{\cancel{n}}\right)\cancel{n} + \frac{27}{n^3}\left(\sum_{i=1}^{n} i^2\right)\right]$$

$$= \lim_{n\to\infty} \left[9 + \frac{\overset{9}{\cancel{27}}}{n^{\cancel{3}\,2}} \frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}\ 2}\right]$$

$$= \lim_{n\to\infty} \left[9 + \frac{9}{2}\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)\right]$$

$$= 9 + \left(\frac{9}{2}\right)(2)$$

$$= 18$$

(2)[5 points] Find

$$I = \frac{1}{6}\int_0^1 6x^2(1+2x^3)^5\,dx$$

$$\left.\begin{aligned} u &= 1+2x^3 \\ du &= 6x^2 dx \end{aligned}\right\} \quad \begin{aligned} x=0 &\Rightarrow u = 1 \\ x = 1 &\Rightarrow u = 3 \end{aligned}$$

$$\therefore I = \frac{1}{6}\int_1^3 u^5\,du$$

$$= \frac{1}{6}\left[\frac{u^6}{6}\right]_1^3$$

$$= \frac{1}{6}\left[\left(\frac{3^6}{6}\right) - \left(\frac{1^6}{6}\right)\right]$$

$$= \frac{3^6-1}{36} = \frac{182}{9}$$

(3)[5 points] Find

$$I = \int \frac{\ln x}{x^2}\,dx$$

$$u = \ln x \;;\; dv = x^{-2}dx$$

$$du = \frac{1}{x}dx \;;\; v = -x^{-1}$$

$$\therefore I = \int u\,dv = uv - \int v\,du$$

$$= (\ln x)(-x^{-1}) - \int (-x^{-1})\left(\frac{1}{x}\right)dx$$

$$= -\frac{\ln x}{x} + \int x^{-2}dx$$

$$= -\frac{\ln x}{x} - \frac{1}{x} + C$$