Math 122 Sec S08N04 - Quiz 2 Name: SOLUTIONS
Jan 21 2008 Stu#:

(1)[**4 points**] Show that for any real number n

$$(\cosh(x) + \sinh(x))^n = \cosh(nx) + \sinh(nx)$$

$$[\cosh(x) + \sinh(x)]^n = \left[\frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2}\right]^n = \left(\frac{2e^x}{2}\right)^n = e^{nx}$$

$$\cosh(nx) + \sinh(nx) = \frac{e^{nx} + e^{-nx}}{2} + \frac{e^{nx} - e^{-nx}}{2} = \frac{2e^{nx}}{2} = e^{nx}$$

$$\therefore [\cosh(x) + \sinh(x)]^n = \cosh(nx) + \sinh(nx).$$

(2)[**3 points**] Find the derivative of

$$H(t) = \tanh(e^t)$$

$$H'(t) = \operatorname{sech}^2(e^t) \cdot e^t$$

$\underline{\underline{or}}$

$$H'(t) = \frac{d}{dt}\left[\tanh(e^t)\right]$$

$$= \frac{d}{dt}\left[\frac{\sinh(e^t)}{\cosh(e^t)}\right]$$

$$= \frac{\cosh(e^t)\cosh(e^t) \cdot e^t - \sinh(e^t)\sinh(e^t)e^t}{\cosh^2(e^t)}$$

$$= \frac{[\cosh^2(e^t) - \sinh^2(e^t)]e^t}{\cosh^2(e^t)}$$

$$= \frac{e^t}{\cosh^2(e^t)} = \operatorname{sech}^2(e^t) \cdot e^t.$$

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(3)[4 points] Find

$$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} \quad \Big\}\ \text{"}\frac{0}{0}\text{"}$$

$$\stackrel{H}{=} \lim_{x\to 0} \frac{e^x - 1}{2x} \quad \Big\}\ \text{"}\frac{0}{0}\text{"}$$

$$\stackrel{H}{=} \lim_{x\to 0} \frac{e^x}{2}$$

$$= \frac{1}{2}$$

(4)[4 points] Find

$$\lim_{x\to 1} \frac{1 - x + \ln x}{1 + \cos(\pi x)} \quad \Big\}\ \text{"}\frac{0}{0}\text{"}$$

$$\stackrel{H}{=} \lim_{x\to 1} \frac{-1 + \frac{1}{x}}{-\sin(\pi x)\cdot \pi} \quad \Big\}\ \text{"}\frac{0}{0}\text{"}$$

$$\stackrel{H}{=} \lim_{x\to 1} \frac{\left(\frac{-1}{x^2}\right)}{-\cos(\pi x)\cdot \pi^2}$$

$$= \frac{-1}{\pi^2}$$