Question 1:

(a)[5 points] Use S_4, Simpson's rule on four subintervals, to approximate $\int_0^4 \sqrt{1+\sin^2(\pi x)}\,dx$.

$f(x) = \sqrt{1+\sin^2(\pi x)}$, $\Delta x = \frac{4-0}{4} = 1$

$S_4 = \frac{1}{3}\left[1\cdot f(0) + 4\cdot f(1) + 2f(2) + 4f(3) + 1\cdot f(4)\right]$

$= \frac{1}{3}\left[1+4+2+4+1\right]$

$\boxed{= 4}$

(b)[5 points] The fourth derivative of $f(x) = \sqrt{1+\sin^2(\pi x)}$ is between -7 and 3 for every x. If we wish to approximate $\int_0^4 \sqrt{1+\sin^2(2\pi)}\,dx$ with accuracy 0.001 using Simpson's rule, how large must n be? Recall, the error in Simpson's rule is at most $\frac{K(b-a)^5}{180n^4}$, where $|f^{(4)}(x)| \le K$ on $[a,b]$.

Here $K = |-7| = 7$.

We want $E_S \le \frac{K(b-a)^5}{180n^4} < 0.001$

$\therefore \frac{7(4-0)^5}{180n^4} < 0.001$

$\therefore \left[\frac{7\cdot 4^5}{180\cdot(0.001)}\right]^{1/4} < n$

$14.1 < n$

$\boxed{\therefore n = 16 \text{ since } n \text{ must be even}}$

Question 2:

(a)[5 points] Evaluate the improper integral $\int_2^3 \frac{1}{\sqrt{3-x}}\,dx$.

$$= \lim_{t \to 3^-} \int_2^t \frac{1}{\sqrt{3-x}}\,dx$$

$$= \lim_{t \to 3^-} \int_2^t (3-x)^{-\frac{1}{2}}\,dx$$

$$= \lim_{t \to 3^-} \left[\frac{(3-x)^{\frac{1}{2}}}{-\frac{1}{2}}\right]_2^t$$

$$= \lim_{t \to 3^-} \left[-2(3-t)^{\frac{1}{2}} + 2(3-2)^{\frac{1}{2}}\right]$$

$$= \boxed{2}$$

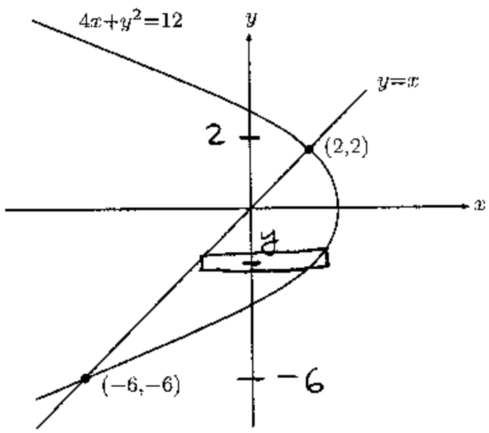
(b)[5 points] Determine if $\int_1^\infty \frac{\sin^2 x}{x^2+\sqrt{x}}\,dx$ converges or diverges. (Do not attempt to evaluate the integral).

$$\frac{\sin^2 x}{x^2+\sqrt{x}} \le \frac{1}{x^2+\sqrt{x}} \le \frac{1}{x^2} \quad \text{on } [1,\infty).$$

$$\therefore \int_1^\infty \frac{\sin^2 x}{x^2+\sqrt{x}}\,dx \text{ converges since } \underbrace{\int_1^\infty \frac{1}{x^2}\,dx}_{\substack{p\text{-integral},\\ p=2.}} \text{ does.}$$

Question 3:

(a)[5 points] Find the area of the region bounded by the curves $4x + y^2 = 12$ and $y = x$.



$$\therefore A = \int_{y=-6}^{2} -\tfrac{1}{4}y^2+3-y \, dy$$

$$= \left[-\tfrac{1}{12}y^3 + 3y - \tfrac{y^2}{2}\right]_{-6}^{2}$$

$$= \left(-\tfrac{8}{12} + 6 - 2\right) - \left(+18-18-18\right)$$

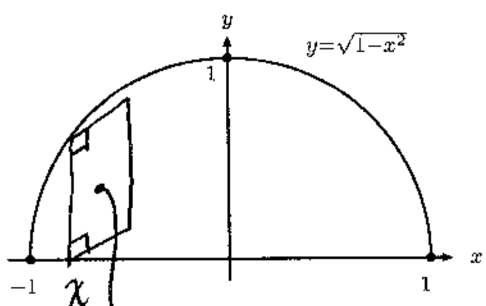
$$= \tfrac{10}{3} + 18$$

$$= \boxed{\tfrac{64}{3}}$$

$4x+y^2 = 12$

$\therefore x = -\frac{1}{4}y^2 + 3$

(b)[5 points] The base of a solid is the region bounded between the curve $y = \sqrt{1-x^2}$ and the x axis. Cross-sections perpendicular to both the base and the x-axis are squares. Find the volume of the solid.



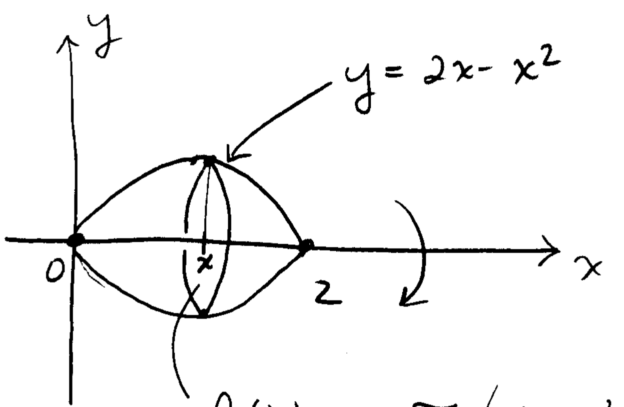
$A(x) = \left(\sqrt{1-x^2}\right)^2 = 1-x^2$

$$\therefore V = \int_{-1}^{1} 1-x^2 \, dx$$

$$= \left[x - \frac{x^3}{3}\right]_{-1}^{1} = \frac{2}{3} - \left(-\frac{2}{3}\right) = \boxed{\frac{4}{3}}.$$

Question 4 [10 points]: The region bounded by the curve $y = 2x - x^2$ and the x axis is rotated about the x-axis. Find the volume of the resulting solid.

$$y = 2x - x^2 = x(2-x)$$



$$A(x) = \pi\,(2x-x^2)^2$$

$$\therefore \quad V = \int_0^2 \pi\,(2x-x^2)^2\,dx$$

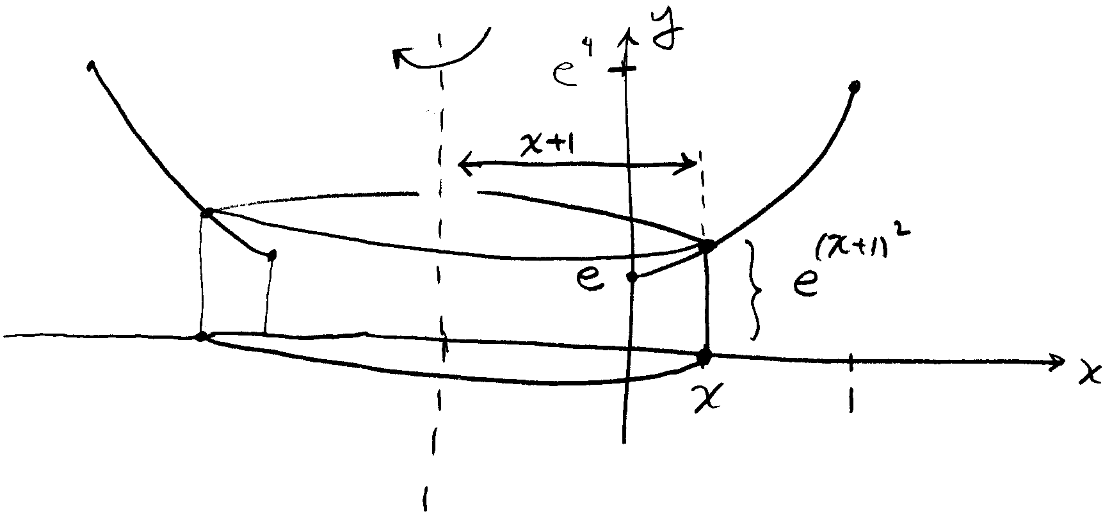
$$= \pi \int_0^2 4x^2 - 4x^3 + x^4\,dx$$

$$= \pi\left[\frac{4}{3}x^3 - \frac{4}{4}x^4 + \frac{x^5}{5}\right]_0^2$$

$$= \pi\left[\frac{32}{3} - 16 + \frac{32}{5}\right]$$

$$= \boxed{\pi\,\frac{16}{15}}$$

Question 5 [10 points]: The region bounded by the curves $y = e^{(x+1)^2}$, $y = 0$, $x = 0$ and $x = 1$ is rotated about the vertical line $x = -1$. Find the volume of the resulting solid.



$$V = \int_0^1 2\pi\,(x+1)\,e^{(x+1)^2}\,dx \qquad \left.\begin{array}{l}\text{let } u = (x+1)^2 \\ du = 2(x+1)\,dx\end{array}\right.$$

$$= \pi\left[e^{(x+1)^2}\right]_0^1$$

$$= \boxed{\pi\left(e^4 - e\right)}$$