Math 122 Sec S08N03/04 - Quiz 6 **Name:** SOLUTIONS
Mar 17 2008 **Stu#:**

(1)[5 points] Evaluate

$$I = \int_1^\infty \frac{\ln x}{x^2}\,dx$$

$$= \lim_{t\to\infty} \int_1^t \frac{\ln x}{x^2}\,dx.$$

$$\int \ln x \; x^{-2}\,dx \quad \Big\}\; \begin{array}{l} u = \ln x \;;\; dv = x^{-2}\,dx \\ du = \frac{1}{x}\,dx \;;\; v = -x^{-1} \end{array}$$

$$= -\frac{\ln x}{x} + \int \frac{1}{x^2}\,dx$$

$$= -\frac{\ln x}{x} - \frac{1}{x}$$

$$\therefore\; I = \lim_{t\to\infty}\left[-\frac{\ln x}{x} - \frac{1}{x}\right]_1^t = \lim_{t\to\infty}\left(-\frac{\ln t}{t} - \frac{1}{t}\right) - \left(\frac{0}{1} - 1\right)$$

$$= 1 - \lim_{t\to\infty} \frac{\ln t}{t}$$

$$\overset{H}{=} 1 - \lim_{t\to\infty} \frac{1/t}{1} = \boxed{1}$$

(2)[5 points] Determine if the following integral converges or diverges (it is easiest to use the Comparison Theorem here):

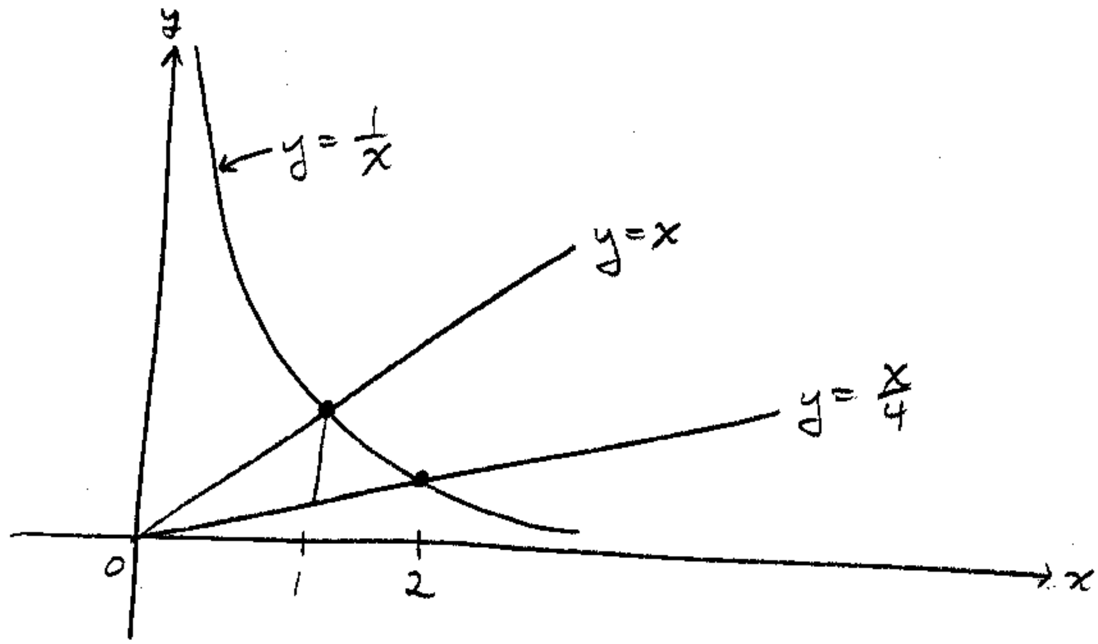
$$\int_1^\infty \frac{1}{x + e^{2x}}\,dx$$

$$\frac{1}{x+e^{2x}} \le \frac{1}{e^{2x}} \quad \text{on } [1,\infty)$$

$$\text{Since } \int_1^\infty \frac{1}{e^{2x}}\,dx = \left[\frac{e^{-2x}}{-2}\right]_1^\infty = \frac{e^{-2}}{2},$$

i.e. $\int_1^\infty \frac{1}{e^{2x}}\,dx$ converges, so does $\int_1^\infty \frac{1}{x+e^{2x}}\,dx$

(3)[5 points] Find the area bounded by $y = 1/x$, $y = x$ and $y = x/4$ where $x > 0$. (Sketch the area in question first).



$\frac{1}{x} = x$ at $x = 1$;

$\frac{1}{x} = \frac{x}{4}$ at $x = 2$

$$\therefore \; A = \int_0^1 x - \frac{x}{4}\,dx + \int_1^2 \frac{1}{x} - \frac{x}{4}\,dx$$

$$= \frac{3}{8}\left[x^2\right]_0^1 + \left[\ln|x| - \frac{x^2}{8}\right]_1^2$$

$$= \frac{3}{8} + \left(\ln 2 - \frac{1}{2}\right) - \left(0 - \frac{1}{8}\right)$$

$$\boxed{= \ln 2}$$