Math 122 Sec S08N03 - Quiz 3
Feb 11 2008

Name: SOLUTIONS
Stu#:

(1)[5 points] Use the definition of the definite integral in the form $\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$ to evaluate

$$\int_0^3 (3+x^2)\,dx\ .$$

Recall that

$$\sum_{i=1}^{n} c = cn \text{ where } c \text{ is a constant}, \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}\ .$$

$$f(x) = 3+x^2$$

$$\Delta x = \frac{3-0}{n} = \frac{3}{n}$$

$$x_i = 0 + i\,\Delta x = 0 + i\left(\frac{3}{n}\right) = \left(\frac{3}{n}\right)i$$

$$\therefore \int_0^3 (3+x^2)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}(3+x_i^2)\Delta x$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(3+\left(\frac{3}{n}i\right)^2\right)\left(\frac{3}{n}\right)$$

$$= \lim_{n\to\infty}\left[\left(\sum_{i=1}^{n}\frac{9}{n}\right)+\left(\sum_{i=1}^{n}\frac{27}{n^3}i^2\right)\right]$$

$$= \lim_{n\to\infty}\left[\left(\frac{9}{n}\right)(n)+\frac{27}{n^3}\left(\sum_{i=1}^{n}i^2\right)\right]$$

$$= \lim_{n\to\infty}\left[9+\frac{\cancel{27}^{\,9}}{n^{\cancel{3}\,2}}\,\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}\,2}\right]$$

$$= \lim_{n\to\infty}\left[9+\frac{9}{2}\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)\right]$$

$$= 9+\left(\frac{9}{2}\right)(2)$$

$$= 18$$

(2)[5 points] Find

$$I = \frac{1}{6}\int_0^1 6x^2(1+2x^3)^5\,dx$$

$$u = 1+2x^3 \qquad x=0 \Rightarrow u=1$$
$$du = 6x^2\,dx \qquad x=1 \Rightarrow u=3$$

$$\therefore I = \frac{1}{6}\int_1^3 u^5\,du$$
$$= \frac{1}{6}\left[\frac{u^6}{6}\right]_1^3$$
$$= \frac{1}{6}\left[\frac{3^6}{6} - \frac{1^6}{6}\right]$$
$$= \frac{3^6-1}{36}$$
$$= \frac{728}{36} = \frac{182}{9}$$

(3)[5 points] Find

$$I = \int \frac{\ln x}{x^2}\,dx$$

$$u = \ln x \; ; \; dv = x^{-2}\,dx$$
$$du = \frac{1}{x}\,dx \; ; \; v = -x^{-1}$$

$$\therefore I = \int u\,dv = uv - \int v\,du$$
$$= (\ln x)(-x^{-1}) - \int (-x^{-1})\left(\frac{1}{x}\right)dx$$
$$= -\frac{\ln x}{x} + \int x^{-2}\,dx$$
$$= -\frac{\ln x}{x} + \frac{x^{-1}}{-1} + C$$
$$= -\frac{\ln x}{x} - \frac{1}{x} + C$$