

(1)[4 points] Show that for any real number n

$$(\cosh(x) + \sinh(x))^n = \cosh(nx) + \sinh(nx)$$

$$[\cosh(x) + \sinh(x)]^n = \left[\frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2} \right]^n = \left(\frac{2e^x}{2} \right)^n = e^{nx}$$

$$\cosh(nx) + \sinh(nx) = \frac{e^{nx} + e^{-nx}}{2} + \frac{e^{nx} - e^{-nx}}{2} = \frac{2e^{nx}}{2} = e^{nx}$$

$$\therefore [\cosh(x) + \sinh(x)]^n = \cosh(nx) + \sinh(nx).$$

(2)[3 points] Find the derivative of

$$H(t) = \tanh(e^t)$$

$$H'(t) = \operatorname{sech}^2(e^t) \cdot e^t$$

$$\begin{aligned} \text{or } H'(t) &= \frac{d}{dt} [\tanh(e^t)] \\ &= \frac{d}{dt} \left[\frac{\sinh(e^t)}{\cosh(e^t)} \right] \\ &= \frac{\cosh(e^t)\cosh(e^t) \cdot e^t - \sinh(e^t)\sinh(e^t)e^t}{\cosh^2(e^t)} \\ &= \frac{[\cosh^2(e^t) - \sinh^2(e^t)]e^t}{\cosh^2(e^t)} \\ &= \frac{e^t}{\cosh^2(e^t)} = \operatorname{sech}^2(e^t) \cdot e^t. \end{aligned}$$

(3)[4 points] Find

$$\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} \quad \left. \vphantom{\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}} \right\} \text{"0/0"}$$

$$\stackrel{H}{=} \lim_{x \rightarrow 0} \frac{e^x - 1}{2x} \quad \left. \vphantom{\lim_{x \rightarrow 0} \frac{e^x - 1}{2x}} \right\} \text{"0/0"}$$

$$\stackrel{H}{=} \lim_{x \rightarrow 0} \frac{e^x}{2}$$

$$= \frac{1}{2}$$

(4)[4 points] Find

$$\lim_{x \rightarrow 1} \frac{1 - x + \ln x}{1 + \cos(\pi x)} \quad \left. \vphantom{\lim_{x \rightarrow 1} \frac{1 - x + \ln x}{1 + \cos(\pi x)}} \right\} \text{"0/0"}$$

$$\stackrel{H}{=} \lim_{x \rightarrow 1} \frac{-1 + \frac{1}{x}}{-\sin(\pi x) \cdot \pi} \quad \left. \vphantom{\lim_{x \rightarrow 1} \frac{-1 + \frac{1}{x}}{-\sin(\pi x) \cdot \pi}} \right\} \text{"0/0"}$$

$$\stackrel{H}{=} \lim_{x \rightarrow 1} \frac{\left(-\frac{1}{x^2}\right)}{-\cos(\pi x) \cdot \pi^2}$$

$$= \frac{-1}{\pi^2}$$