

**Math 192**  
**Practice Problems for Test 3**

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The following problems are good practice for the upcoming test. Some are challenging, but completely within the scope of the material we have covered.

1. If  $y$  is a solution to the differential equation  $\frac{dy}{dx} = x(2y + 1)^2$  where  $y(0) = 0$ , find  $y(1/2)$ .

9/1 :ans

2. Compute  $\int_0^\infty \frac{1}{x^2 + 4x + 3} dx$ .

8 1/2 :ans

3. Let  $A(t)$  be the total debt of a company in millions of dollars. If debt is subject to a continuous interest rate 6% per year while payments on the debt are made continuously at the rate of 1/2 million dollars per year, the debt can be modeled by the differential equation

$$\frac{dA}{dt} = 0.06A - 1/2.$$

If the debt is currently \$2,000,000, how many years will it take for the debt to be paid off?

ans: 50 ln(61/25) = 4.6 years

4. A business generates profits at the rate of  $P(t) = 3e^{0.02t}$  where  $P$  is measured in units of millions of dollars per year. You purchase the business for 48 million dollars, this amount being the present value of all future profits. What (constant) continuously compounded interest rate is being used to determine the present value?

ans: 8.25%

5. Suppose  $f(0) = 6$ ,  $f(1) = 5$  and  $f'(1) = 2$ . Compute  $\int_0^1 x f''(x) dx$ .

3 :ans

6. Compute the following integrals:

(i)  $\int 27x^2 (\ln(x))^2 dx$ .

ans:  $9x^3 \ln(x) - \frac{9}{2}x^3 + \frac{27}{2}x^3 \ln^2(x) + C$

(ii)  $\int (t+2) \sqrt{2+3t} dt$ .

ans:  $\frac{2}{5}(3t+2)^{5/2} - \frac{2}{3}(3t+2)^{3/2} + \frac{4}{15}(3t+2)^{1/2} + C$

(iii)  $\int \frac{a-b}{(x-a)(x-b)} dx$ .

ans:  $\ln \left| \frac{a-x}{b-x} \right| + C$

7. The weekly revenue generated by a new product is given by  $S(t) = 1000te^{-t} + 1000$  (in dollars per week), where  $t = 0$  is the present. What is the average weekly revenue over the first four weeks?

ans: approx. \$1227

8. What is  $k$  if the average value of  $y = x(x - 1)$  over the interval  $[0, k]$  is  $k$ ?

 $7/6$  :sure

9. Recall the differential equation describing Newton's law of cooling:

$$\frac{dT}{dt} = k(T - a) .$$

If a pizza removed from a 170 degree oven is found to have a temperature of 160 after five minutes of sitting in a 20 degree room, how long will it take for the pizza to cool to 100 degrees?

ans: approx. 46 min.

10. A retiring college professor has \$300,000 in his retirement fund which pays interest at 5% per year compounded continuously. He will make no further deposits to the fund. He would like to withdraw  $P$  dollars per year starting now, and increase this withdrawal rate continuously by 2.5% per year to account for price inflation. What should  $P$  be so that all funds are exhausted at the end of 20 years?

ans: approx. 19,061 dollars per year.