The following problems are good practice for the upcoming test. Some are challenging, but completely within the scope of the material we have covered.
Remember: **the integral of a rate of change gives total change!**

1. Two populations are growing according to different rates. At time t years, the first is growing at $1 + t/2$ critters per year, while the second is growing at $(1/2)(t + t^2/7)$ critters per year. If at time zero both populations are the same size, at what time in the future are they again the same size?

2. Determine the area enclosed by the curves $y = 1 - x^2$ and $y = x$.

3. Compute the following integrals:

   (a)
   $$\int_e^\pi e - \pi + x\,dx$$

   (b)
   $$\int_4^9 \frac{x^2 + \sqrt{x}}{\sqrt{x}}\,dx$$

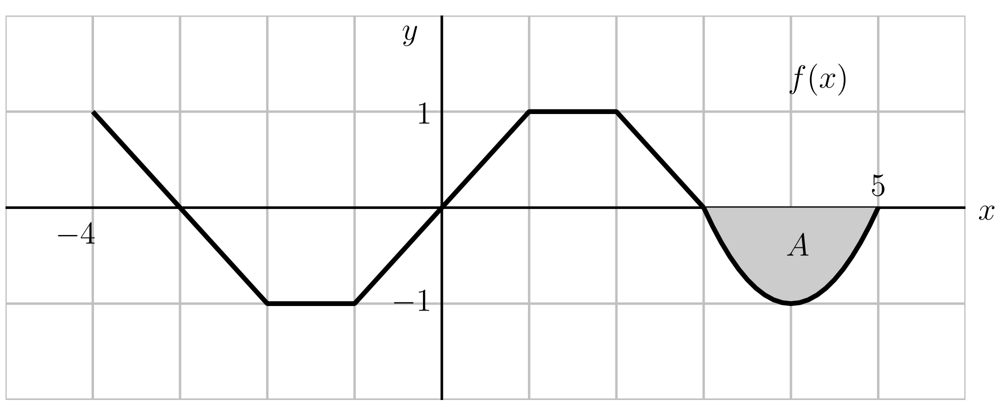
   (c)
   $$\int_0^{a^2} (2x^2 + x)^2\,dx$$

4. Let W be the weight of an animal in kilograms and let t be time in weeks, where $t = 0$ is the present. The rate of change of weight with respect to time, $W'(t)$, in kilograms per week is given by the following table:

t	0	0.5	1.0	1.5	2.0	2.5
$W'(t)$	3	2	3	5	4	2

   If the animal is currently 20 kg, use the trapezoid rule to estimate the weight of the animal in 2.5 weeks.

5. Answer the questions below with reference to the following graph of $f(x)$, $-4 \leq x \leq 5$:



(a) Assuming the area of the shaded region A is $4/3$, what is the exact value of $\int_{-4}^{5} f(x)\,dx$?

(b) Recall from the Fundamental Theorem of Calculus, $F(x) = \displaystyle\int_{-4}^{x} f(t)\,dt$ is an antiderivative of $f(x)$, and notice that $F(0) = 0$. What are the x coordinates of the relative (or local) maxima of $F(x)$ for $-4 < x < 5$?

(c) Again for $F(x)$ the antiderivative described in part (b), on what intervals is $F(x)$ increasing?

6. The population P of a town is now 30,000 and is predicted to grow at an instantaneous rate of

$$P'(t) = \sqrt{\frac{40{,}000}{t+1}} + \left(\frac{5000}{t+1}\right)^{2/3}$$

where t is time in years and $t = 0$ is the present. If the prediction is accurate, what will the population be 3 years from now?

7. Evaluate

$$\int_0^e \frac{(\pi+1)x^\pi - (e+1)x^e}{\pi^2 - e^2}\,dx\ .$$

8. The temperature T in an oven is changing at an instantaneous rate of $18t\sqrt{3t^2+4}$ degrees per minute, where $t = 0$ is the present. If the temperature one minute from now is 40 degrees, what is the temperature two minutes from now?

9. The price of a share of stock is presently \$60 and is changing at an instantaneous rate of $S'(t) = -\dfrac{40t}{t^2+1}$ for $t \geq 0$, where S is the price of the stock in dollars, t is time in months, and $t = 0$ represents the present. At what time will the stock be worthless? Report your answer to three significant figures.

10. Compute the following indefinite integrals:

(a)

$$\int x^{-1} \ln\left(x^3\right) dx$$

(b)

$$\int \frac{x-1}{x^2 - 2x + 5}\,dx$$

(c)

$$\int \frac{x}{(x+1)^2}\,dx$$

11. A mining company wishes to search for a certain mineral by drilling vertical test holes in the ground at a remote mining site. It becomes more expensive to drill the deeper you go into the ground: at a depth of x metres it costs $\dfrac{x\sqrt{x+1}}{4}$ dollars per metre to drill. In addition, it costs \$50,000 to transport the machinery to the mine site and set it up. What is the total project cost to the company if they wish to drill seven test holes of 50 metres each at the mine site?

12. Suppose a copper mine closes and the population N of a nearby town decreases at a rate given by $N'(t) = \dfrac{-600t}{\sqrt{2t^2+16}}$, where t is time in years and $t = 0$ corresponds to the present. Assume that the present population of the town is 8000. How long will it take for the population to reach 5600?

13. Ice is forming on a pond at a rate given by

$$\frac{dy}{dt} = k\sqrt{t},$$

where y is the thickness of the ice in inches at time t measured in hours since the ice started forming, and k is a positive constant.

   (a) Find a formula for y as a function of t (your formula may have k in it, but should contain no other constants).

   (b) If the ice is 2 inches thick after 20 hours, what is the value of the constant k?

14. The supply function for a product is given by the following table, where p is the price per unit (in dollars) at which q units are supplied to the market:

q	0	10	20	30	40	50
p	38	59	69	75	80	84

Use the trapezoid rule to estimate the producers' surplus if the equilibrium selling price is \$80.

15. The weekly demand for a certain substance is given by $p = D(q) = \dfrac{400}{0.5q+2}$. Price is measured in dollars and quantity in grams. The supply function is not known. The equilibrium price is known to be \$20 a gram. Calculate the consumers' surplus, to the nearest cent.

16. Evaluate $\displaystyle\int_1^2 2x^2 - 3x\,dx$ using the definition of the definite integral. That is, approximate the integral using n rectangles, and then take the limit as $n \to \infty$. Recall, $\sum_{j=1}^{n} j = n(n+1)/2$, and $\sum_{j=1}^{n} j^2 = n(n+1)(2n+1)/6$

17. Find the area bounded by the curves $y = 2x^3 - x$ and $y = x^2$.