Math 192 Sec S07N01
Practice Problems for Test 1

1. Let $f(x,y) = 2x^2 - 3xy + 4y^2 + ax + by$, where a and b are constants. If $f(x,y)$ has a relative maximum at $x = 2$, $y = 3$, find a and b.

   ans: $(a,b) = (1, -18)$

2. Let $f(x,y) = \ln(x^2 + y^2)$. Compute and simplify $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2}$.

   ans: 0

3. Let $f(x,y) = xy - x^4/2 - y^2$. Where does $f(x,y)$ attain relative maxima?

   ans: $(1/2, 1/4)$, $(-1/2, -1/4)$

4. A factory is filling an order for 120,000 shirts. Using x units of labour and y units of capital, the factory can produce $x^{2/3}y^{1/3}$ shirts. Labour is measured in person-hours, which cost \$16 each, while capital is measured in dollars. Use the method of Lagrange multipliers to find the cost of producing the sweatshirts at minimum cost. (You may assume the constrained critical point you find gives the minimum cost.)

   ans: $x = 60,000$, $y = 480,000$ for minimum cost of $\$1,440,000$

5. A manufacturer sells the same product in two markets: A and B. x tons of product can be sold in market A at a price of $13000 - 5x$ dollars per ton, while y tons of product can be sold in market B at a price of $9000 - y$ dollars per ton. Manufacturing and shipping costs for either market is 1000 dollars per ton. Find the number of tons of product that should be sold in each market in order to maximize total profit.

   ans: $x = 1200$, $y = 4000$

6. A greenhouse tomato grower finds that spending x dollars per week on electricity and y dollars per week on fertilizer yields $h(x,y)$ kilograms of tomatoes, where $h(x,y) = x^{4/5}y^{1/5}$. The grower must produce 100 kilograms per week for his buyers. Use the method of Lagrange multipliers to calculate the minimum weekly cost of producing the required number of tomatoes. (You may assume the constrained critical point you find gives the minimum.)

   ans: \$164.94

7. Let $f(x,y) = (x^3 - 3x + 3)(y^2 + 1)$. Find all points (x,y) where $f(x,y)$ attains a local maximum, all points where $f(x,y)$ attains a local minimum, and all saddle points, i.e. critical points at which $f(x,y)$ attains neither a local maximum nor local minimum. Note: in reference to maxima and minima, the word "local" means the same "relative".

   ans: loc. min at $(1,0)$; saddle point at $(-1,0)$

8. Let $F(K,L) = 30K^{1/3}L^{2/3}$ (called a Cobb-Douglas production function). Given that $K/L = 2$, find the marginal productivity of labour $F_L(K,L)$.

   ans: $20\sqrt[3]{2}$

9. If the daily available number of doctor-hours in an emergency ward is x and the number of nurse-hours is y, the satisfaction level of patients is $S(x, y)$, where

$$S(x, y) = \frac{xy}{x + y}$$

The administrator wants to maximize patient satisfaction, but can only spend a combined total of \$1600 a day on doctors and nurses. Doctors are paid \$128 per hour and nurses are paid \$8 per hour. How many doctor-hours and how many nurse-hours should the administrator schedule per day? Use the method of Lagrange multipliers, and assume the constrained critical point you find yields maximum satisfaction.

ans: 10 doctor-hours, 40 nurse-hours

10. You have an exclusive franchise to sell a nutritional supplement at your two stores A and B. You practice price discrimination by charging different prices in the two stores. If you charge p_A dollars per ounce in store A, and p_B dollars in store B, the daily sales in store A are q_A ounces and daily sales in store B are q_B ounces, where

$$q_A = 80 - 2p_A + p_B \text{ and } q_B = 70 - 2p_B \ .$$

You can buy any quantity of the supplement from your wholesaler for \$10 per ounce. At what price should you sell the supplement in store A, and at what price in store B, in order to maximize profit? (You may assume the critical point you find yields maximum profit).

ans: A: 32; B:28

11. Practice computing partial derivatives: any of problems 17.2:1-34 of the text.