

Math 192 Sec S07N01 Final Exam – Apr 18 2007

name (printed)

student number

I have read and understood
the instructions below:

signature

Instructions:

1. No notes or books are to be used in this exam. If you need scrap paper please ask and some will be provided.
2. A non-programmable, non-graphing calculator is permitted.
3. There are 12 pages (including this cover page) in the exam. Justify every answer, and show your work. Unsupported answers will receive no credit.
4. You will be given three hours to write this exam. Read over the exam before you begin.
5. At the end of the three hours you will be given the instruction to stop writing. *Continuing to write after this instruction will be considered as cheating.*
6. **Academic dishonesty:** Exposing your paper to another student, copying material from another student, or representing your work as that of another student constitutes academic dishonesty. Cases of academic dishonesty may lead to a zero grade in the exam, a zero grade in the course, and other measures, such as suspension from this university.

Question	value	score
1	45	
2	10	
3	10	
4	10	
5	8	
6	7	
7	10	
Total	100	

Question 1: [45 points] This question consists of 15 short answer problems each worth 3%. For each problem, clearly write your final answer in the box to the right. The solution to each problem is short, requiring no more space than that given.

(a) Let $f(x, y) = \sqrt{xy}e^{2+x}$. Compute $f_x(1, 1)$.

(b) A company's joint cost function for two products is $c = 0.002(x + y)^2 + x + 0.25y + 8000$, where x and y represent the number of units of product X and product Y , respectively. What is the marginal cost with respect to product X when $x = 450$ and $y = 550$?

(c) Find the critical point of $f(x, y) = x^2 + 2xy + 2y^2 - 4y$.

- (d) For a certain production function $P(\ell, k)$, $P(100, 200) = 350$, $P_\ell(100, 200) = -5$ and $P_k(100, 200) = 7$. Estimate the production if ℓ is decreased by 2 while k is kept at 200.

- (e) Compute $\int 5x^4 - \frac{2}{x^2} dx$.

- (f) Marginal cost is given by $\frac{dc}{dq} = 3q + q^3 + e^q$ while fixed costs are \$100. Find the cost function $c(q)$.

(g) Evaluate $\int \frac{t}{\sqrt{4-9t^2}} dt$.

(h) The demand equation for a product is $p = 200 - q^2$, while supply is given by $p = 6q + 160$. The equilibrium quantity is $q = 4$. Find the producer surplus.

(i) Compute $\int \frac{x}{e^{5x}} dx$.

(j) Compute $\int \frac{x}{(x+1)(3-x)} dx$.

(k) The average value of $f(x) = \frac{1}{1+x}$ over the interval $[0, k]$ is $1/k$ for some value of $k > 0$. Determine k .

(l) Solve the differential equation $\frac{dy}{dx} = e^{x+2}y^2$ where $y(-2) = -1/2$.

(m) Compute $\int_0^\infty x e^{-x^2} dx$.

(n) Let $\mathbf{A} = \begin{bmatrix} 2 & -1 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 3 & -5 \\ -9 & 2 \end{bmatrix}$, and $\mathbf{C} = \begin{bmatrix} -2 & 2 \\ 4 & -1 \end{bmatrix}$. Compute $\mathbf{A}(\mathbf{B} + 2\mathbf{C})$.

(o) Suppose \mathbf{A} is size 4×1 , \mathbf{B} is size 6×4 , \mathbf{C} is size 4×4 , and \mathbf{D} is size 1×6 . What is the size of the product $DBCA$?

Question 2: [10 points]

A company's product has production function $P = 100\ell + 50k - \ell^2 - k^2$ where P is the number of units of output resulting from ℓ units of labour and k units of capital. Labour costs \$600 per unit, while capital costs \$300 per unit. Use the method of Lagrange multipliers to determine the maximum possible output if the total cost of labour and capital is to be \$3000. (You may assume that the critical point you find does indeed yield the desired maximum).

Question 3: [10 points]

Suppose you retire at age 65 and you are offered two options for receiving your continuously paid pension income:

- (i) \$25,000 per year, paid continuously, or
- (ii) payments made continuously at the rate of $p(t) = 16,000e^{0.04t}$ dollars per year, where $t = 0$ corresponds to day you begin receiving payments at age 65.

You plan to (continuously) deposit your pension payments into a fund which pays 8% interest, compounded continuously, and at age 85 you will give the accumulated value of the investment fund to your grandchildren as a gift. Which of the two options should you choose to yield the larger gift?

Question 4: [10 points]

Use the trapezoid rule with four sub-intervals to estimate the area in the first quadrant bounded by the curve $y = \sqrt{8 - x^3}$, the x -axis and the y -axis. Round your answer to two decimal places.

Question 5: [8 points]

The population of a certain town is currently 10,000 and is growing the rate of $P'(t) = \frac{1500 \ln(1 + 2t)}{1 + 2t}$ people per year, where t is measured in years and $t = 0$ corresponds to the present. How long will it take for the population to reach 12,000?

Question 6: [7 points]

As a result of competition, a phone company is losing customers according to the model

$$\frac{dA}{dt} = -k\sqrt{A}$$

where $A(t)$ is the number of customer accounts at time t years, and $t = 0$ corresponds to the present. If there are currently 490,000 customers, and projections forecast a decrease of 67,500 customers over the next year, determine k and find an expression for $A(t)$.

Question 7: [10 points]

Solve the following system of equations using matrix reduction (or equivalently, Gauss-Jordan elimination). Clearly write the elementary row operation you are using at each stage of the reduction, and clearly state your final solution.

$$2x - 3y + 2z = 2$$

$$x + 4y - z = 9$$

$$-3x + y - 5z = 5$$