Question 1: [**40 points**] This question consists of 20 short answer problems each worth 2%. For each problem, clearly write your answer in the box to the right AS ONLY THAT ANSWER WILL BE GRADED. The solution to each problem is short, requiring no more space than that given. Although no part marks are awarded, show your work clearly in case it is needed to support your final answer.

(a) If $z = f(x, y) = \dfrac{3x + 2y^2}{xy}$, compute $\left.\dfrac{\partial z}{\partial x} - \dfrac{\partial z}{\partial y}\right|_{(2,1)}$.

(b) Find the critical points of $g(p, q) = p^2 + p \ln q$.

(c) If demand for products A and B are given by $q_A = 20 - p_A - 2p_B$ and $q_B = 50 - 2p_A - 3p_B$, determine if these products are competitive, complementary, or neither.

(d) Let $P(l, k) = 2k^2 + 8l$ be a production function, where k is capital and l is labour. For what value of k is marginal productivity with respect to capital equal to marginal productivity with respect to labour?

(e) If $f_x(-2, 1) = f_y(-2, 1) = 0$, while $f_{xx}(-2, 1) = -3$, $f_{yy}(-2, 1) = -2$, and $f_{xy}(-2, 1) = 1$, determine if $f(-2, 1)$ is a relative maximum, minimum or neither.

(f) Compute $\displaystyle\int \frac{2x^{1/2} - 3x^{1/3}}{x}\, dx$.

(g) Given the marginal revenue function $\dfrac{dr}{dq} = 20 - q - q^2$, find the demand equation.

(h) Suppose $y' = \dfrac{x^2 + 1}{2x^3 + 6x + 1}$ and $y(0) = 1$. Find y as a function of x.

(i) Compute $\displaystyle\int \frac{\ln(2xe^x)}{x}\, dx$.

(j) If marginal cost is given by $\frac{dc}{dq} = 200 - q - q^{-1/2}$, find the increase in cost if production is increased from 25 to 36 units.

(k) The supply and demand functions for a product are $q = p/2 - 10$ and $q = \sqrt{100 - p}$, respectively. Determine the producer surplus for the product.

(l) Find $\int \frac{1}{x^2(x-2)}\, dx$.

(m) Evaluate $\int \sqrt{x} \ln x \, dx$.

(n) Population is modeled by the function $P(t) = 5000e^{0.05t}$, where t is time in years and $t = 0$ is the present. What is the average population over the next five years?

(o) Solve the differential equation $x^2 \dfrac{dy}{dx} = e^{-y/2}$, $y(1) = 0$.

(p) Determine if $\int_0^\infty \frac{1+e^x}{x+e^x}\,dx$ converges.

(q) Let $\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} -1 & 3 \\ -2 & -4 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$. Compute $(\mathbf{A} - 2\mathbf{B})^{\mathrm{T}}\mathbf{C}$.

(r) The matrix $\begin{bmatrix} 1 & -2 \\ -3 & 6a \end{bmatrix}$ is not invertible. What is a?

(s) If $\mathbf{A}$ is size 3×2 and $\mathbf{C}$ is size 3×5, what size is $\mathbf{B}$ in order for $\mathbf{ABC}$ to be defined?

(t) Solve $\mathbf{A}\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 12 \\ 15 \end{bmatrix} = \mathbf{0_{2\times 1}}$ given that $\mathbf{A^{-1}} = \begin{bmatrix} 1/2 & -1/3 \\ -1/4 & 1/5 \end{bmatrix}$.

Question 2: [**10 points**]
The price of a standard house in the Nanaimo region is increasing according to the model

$$P'(t) = \frac{P_0 t}{\sqrt{16 + t^2}} ,$$

where $P(t)$ is standard house price at time t, and $P_0 = P(0)$. How long will it take a for the price of a standard house to double?

Question 3: [10 points]
A factory manufactures products A and B. The production level is measured in units of 1000 items. If the factory manufactures and sells x units of product A and y units of product B, the resulting profit (in tens of thousands of dollars) is

$$P(x, y) = 6x + 9y - x^2 - y^2 - xy - 4 \ .$$

(a) [**8 points**] Find the number of units of A and B which should be manufactured to maximize profit. Verify that the values you give do indeed give the maximum.

(b) [**2 points**] Find the resulting maximum profit in dollars.

Question 4: [**10 points**]
Suppose a college professor makes continuous contributions to a retirement fund over a ten year period at the rate of $f(t) = 10000(1 + 0.1t)$ dollars per year. Here t is in years, and $t = 0$ is the present.

(a) [**4 points**] Find the average contribution rate over the ten year period.

(b) [**6 points**] Find the accumulated value of the contributions at the end of the ten year period if the retirement fund earns interest at 7% interest compounded continuously.

Question 5: [10 points]
Data for the supply and demand functions $S(q)$ and $D(q)$ for a certain product is available as follows:

q	0	1	2	3	4	5
$S(q)$	1.0	2.3	3.1	3.5	4.1	4.5
$D(q)$	8.0	7.6	6.5	5.9	5.4	4.5

Use the trapezoid rule to estimate the total of the consumer and producer surplus.

Question 6: [10 points]
Let

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 2 & 0 \\ 3 & -2 & 2 \end{bmatrix}$$

(a) [**7 points**] Find $\mathbf{A}^{-1}$.

(b) [**3 points**] Solve the following system of equations

$$\begin{aligned} 2x + y &= -3 \\ 3x + 2y &= 2 \\ 3x - 2y + 2z &= -2 \end{aligned}$$

(Your result from part (a) may help here.)

Question 7: [10 points]
A retiring college professor has \$500,000 in savings in an account earning 6% interest compounded continuously. If he continuously withdraws money at the rate of k dollars per year, the amount $A(t)$ of money in the fund at time t obeys the differential equation

$$\frac{dA}{dt} = 0.06A - k \ , \qquad A(0) = \$500{,}000 \ .$$

(a) [**4 points**] If $k = \$60{,}000$, solve the differential equation to find $A(t)$.

(b) [**4 points**] Again with $k = \$60{,}000$, how long will it take for balance of the saving account to reduce to zero?

(c) [**2 points**] For what value of k will the amount of money in the account remain at a constant \$500,000 forever? (this is an easy question if you think about right– think about the derivative of $A(t)$.)