

(1)[5 points] Let $z = \ln(x^2 + y^2)$. Show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0 .$$

(2)[5 points] Find the critical points of $f(x, y) = y - y^2 - 3x - 6x^2$.

(3)[5 points] An open-top rectangular box is to have a volume of 6 m^3 . The cost per square metre of material is \$3 for the bottom, \$1 for the front and back, and \$0.50 for the other two sides. Find the dimensions of the box so that the cost of materials is minimized. (Show that the critical point you find corresponds to a relative minimum. You may assume that it also gives the absolute minimum.)