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Math 110 Sec S07N01 Final Exam – Apr 18 2007

name (printed)

student number

I have read and understood the instructions below:

signature

Instructions:

1. No notes or books are to be used in this exam. A list of formulas can be found on the last page. If you need scrap paper please ask and some will be provided.
2. A non-programmable, non-graphing calculator is permitted.
3. There are 12 pages (including this cover page) in the exam. Justify every answer, and show your work. Unsupported answers will receive no credit.
4. You will be given three hours to write this exam. Read over the exam before you begin.
5. At the end of the three hours you will be given the instruction to stop writing. *Continuing to write after this instruction will be considered as cheating.*
6. **Academic dishonesty:** Exposing your paper to another student, copying material from another student, or representing your work as that of another student constitutes academic dishonesty. Cases of academic dishonesty may lead to a zero grade in the exam, a zero grade in the course, and other measures, such as suspension from this university.

Question	value	score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	**100**	

Question 1:

(a)[3 points] Determine the convergence or divergence of the sequence $a_n = \dfrac{\pi n}{\sqrt{4n^2 - n}}$.

(b)[3 points] Determine the convergence or divergence of the sequence $a_n = \dfrac{(n+2)!}{n!\sqrt{n}}$.

(c)[4 points] The sequence $c_n = (a_n - b_n)$ converges to $1/2$, where $a_n = \dfrac{n-1}{n} + \dfrac{n+1}{n}$. What is $\lim_{n\to\infty} b_n$?

Question 2: Find the sum of the following convergent series:

(a)[5 points] $$\sum_{n=0}^{\infty} \left(\frac{-3\pi}{11}\right)^n$$

(b)[5 points] $$\sum_{n=0}^{\infty} \frac{1}{(2n+1)(2n+3)}$$

Hint: $\dfrac{1}{(2n+1)(2n+3)} = \dfrac{1}{2}\left(\dfrac{1}{2n+1} - \dfrac{1}{2n+3}\right)$.

Question 3: Determine the convergence or divergence of the following series:

(a)[5 points] $$\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^2+1}.$$

(b)[5 points] $$\sum_{n=1}^{\infty} \frac{n^3-3n}{3n^3+\sqrt{n}}.$$

Question 4: Determine the convergence or divergence of the following series:

(a)[5 points] $$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!}.$$

(b)[5 points] $$\sum_{n=1}^{\infty} \frac{5^n n}{(n-1)!}.$$

Question 5:

(a)[5 points] Determine the radius of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}(x+1)^n}{2^n n!}$.

(b)[5 points] Find a power series centered at $c = -2$ for $f(x) = \frac{2}{2-3x}$ and state its radius of convergence.

Question 6:

(a)[5 points] Let $f(x) = e^{\cos x}$. Use a Maclaurin polynomial of degree two to approximate $f(0.1)$.

(b)[5 points] Recall that $\int \frac{1}{1+x^2}\,dx = \arctan(x)$. Use this fact to find a power series centered at 0 for $\arctan(x)$ and state its radius of convergence.

Question 7:

(a)[6 points] The function $\sinh(x)$ (called the hyperbolic sine function) is defined by

$$\sinh(x) = \frac{e^x - e^{-x}}{2} .$$

Find the first three non-zero terms of the Maclaurin series for $\sinh(x)$.

(b)[4 points] Use your result in (a) to find

$$\lim_{x \to 0} \frac{\sinh(x^2)}{x^2} .$$

Question 8: Consider the parametric curve defined by

$$x = 1 + 3t^2 \ , \qquad y = 4 + 2t^3$$

(a)[6 points] Find the arc length of the curve over the t-interval $[0, 1]$.

(b)[4 points] Find the equation of the tangent line to the curve at the point where $t = 1$.

Question 9:

(a)[6 points] Carefully sketch the the polar curve

$$r = 2 - 2\sin(\theta)$$

and label the points corresponding to $\theta = \pi/4$ and $\theta = 3\pi/2$.

(b)[4 points] Write an integral which gives the arc length of the portion of the curve lying in quadrants one and two. DO NOT EVALUATE THE INTEGRAL!

Question 10 [10 points]: Again using the polar curve

$$r = 2 - 2\sin(\theta) \ ,$$

find the area of the region enclosed by the curve. You may find the trigonometric identities listed on the formula sheet useful in the evaluation of the resulting integral.

You may find some of the following useful:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots, \quad |x| < 1$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, \quad |x| < 1$$

$$\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\sin\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$$