Question 1: [**10 points**]

(a)[**4 points**] Determine if the sequence with terms $a_n = \sqrt{n} - \sqrt{n-1}$, $n = 1, 2, 3, \ldots$, converges or diverges.

(b)[**4 points**] Determine if the sequence with terms $b_n = \dfrac{n^2 + n^{1/2}}{2n^{1/2}}$, $n = 1, 2, 3, \ldots$, converges or diverges.

(c)[**2 points**] What is the smallest positive value of r for which the sequence with terms $a_n = (\pi/r)^n$ converges?

Question 2: [10 points]

(a)[**5 points**] Find the sum of the series $\displaystyle\sum_{n=1}^{\infty}\left[\frac{1}{n+1}-\frac{1}{n+2}\right]$.

(b)[**5 points**] Find the sum of the series $\displaystyle\sum_{n=0}^{\infty}\left[\left(\frac{-2}{3}\right)^{n}-\frac{3}{2^{n}}\right]$.

Question 3: [10 points]

(a)[5 points] Determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{2n-5}$ converges or diverges.

(b)[5 points] Determine if the series $\sum_{n=0}^{\infty} \frac{\pi^n}{2^{2+n} e^n}$ converges or diverges.

Question 4: [10 points]

(a)[5 points] Determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n6^n}$ converges or diverges.

(b)[5 points] Determine if the series $\sum_{n=1}^{\infty} \frac{(n+1)^5 5^{n+1}}{n^7 7^n}$ converges or diverges.

Question 5: [10 points]

(a)[5 points] Construct the Taylor polynomial of degree 2 centered at $x = 2$ for the function $f(x) = (4 + 2x)^{1/3}$.

(b)[5 points] Let $T_2(x)$ be the polynomial you constructed in part (a). If $T_2(x)$ is used to approximate $f(3)$, use the error term from Taylor's Theorem to give a bound on $|f(3) - T_2(x)|$.

Question 6: [10 points]

(a)[5 points] Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{\pi^n (x-5)^n}{n}$.

(b)[5 points] Find a power series centered at $x = 0$ for $f(x) = \frac{5}{7+x}$.

Question 7: [10 points]

(a)[5 points] Using the fact that

$$\frac{d}{dx}\left[\frac{1}{(1+x^2)}\right] = \frac{-2x}{(1+x^2)^2} \ ,$$

find a power series centered at $x = 0$ for $\frac{-2x}{(1+x^2)^2}$.

(b)[5 points] Find the first four terms of the Maclaurin series for $f(x) = 2x^3 \ln(1 - x^2)$.

Question 8: [10 points]

Consider the parametric equations

$$\left.\begin{aligned} x &= 3t^2 - 5 \\ y &= t + \frac{4}{t} \end{aligned}\right\} \qquad \text{where } t \neq 0.$$

(a)[3 points] Eliminate the parameter t to write the corresponding equation in rectangular coordinates. (Squaring the expression for y as a first step may help here.)

(b)[3 points] Find the points (x, y) where tangent lines to the curve are horizontal.

(c)[4 points] Find the equation of the tangent line to the curve at the point where $t = -1$.

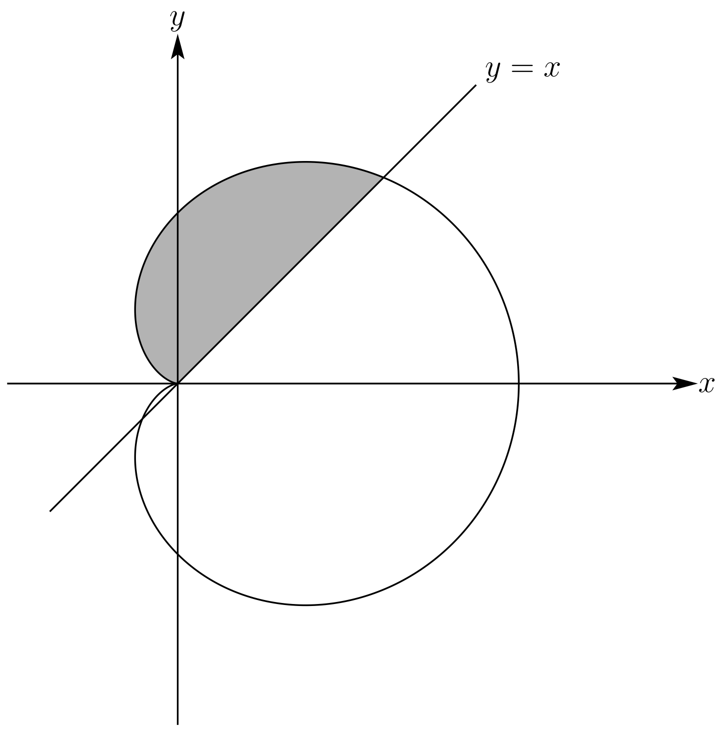
Question 9: [10 points]

(a)[**5 points**] Carefully sketch the polar curve $r = 3\sin(2\theta)$. Show on your sketch the polar coordinates (r, θ) of at least 4 points.

(b)[**5 points**] Write an integral which gives the arc length of the portion of the curve in part (a) located in the first quadrant. Do not evaluate the integral.

Question 10: [10 points]

The polar curve $r = 2 + 2\cos\theta$ and the line $y = x$ are shown in the plot below:



Determine the area of the shaded region.

You may find some of the following useful:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots\,, \qquad |x| < 1$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots\,, \qquad |x| < 1$$

$$\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\sin\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$