Math 152 Sec S0601/S0602
Practice Problems for Final Exam

The following problems are good practice for the final exam. Some of these problems are challenging but completely within the scope of the material we have covered. The problems on the final exam may of course be completely different.

Shorter Problems

1. Let $f(x) = \sqrt{x+5} - 2x$. Compute and simplify $\dfrac{f(x+h) - f(x)}{h}$.

2. Simplify $\dfrac{6a^{1/2}b^3c - 12a^2b^{1/3}}{4a^{1/2}b^{1/3}c}$.

3. Solve $\dfrac{2}{x} = \dfrac{7}{x^2} - 3$ for x.

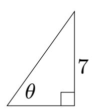
4. Solve $t^{2/3} + t^{1/3} - 4 = 0$ for t.

5. Convert π/a radians to degrees.

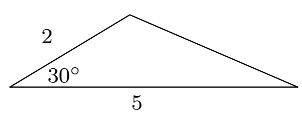
6. Find $\cos(95\pi/6 - 10\pi)\tan(-50\pi/3)$ exactly.

7. If $\sin\alpha = 2/\sqrt{7}$ and $\pi/2 \leq \alpha \leq \pi$, find $\tan\alpha$.

8. If $\cos\alpha = a/b$, what is $\csc^2\alpha$?

9. If $\sec\theta = \sqrt{5}$ in 

find all remaining sides and angles. (Note figure not to scale).

10. Find all remaining sides and angles of 

(Note figure not to scale).

11. Triangle ABC has angle $B = 25°$, the side opposite B is $b = 7$, and the angle opposite angle C is $c = 13$. Find all possible values for the remaining sides and angles.

12. Given that $\sin(\pi/10) = (\sqrt{5} - 1)/4$, find $\sin(\pi/5)$ exactly.

13. If $\sin\theta = x/4$ where $0 \leq \theta \leq \pi/2$, find an expression for $\sin(2\theta)$ which does not involve trigonometric functions.

14. Compute $\tan[\sin^{-1}(-5/7)]$.

15. Simplify $\sin^{-1}[\sin(-81\pi/10)]$.

16. Compute $(3!)!/(3!)^2$.

17. Compute and simplify $(1/2 + 2a)^6$.

18. Suppose the matrix $\begin{bmatrix} 2 & -3 \\ k & 3 \end{bmatrix}$ is not invertible. What is k?

19. Suppose $\mathbf{A}$ and $\mathbf{B}$ are column matrices with 5 entries each. What is the size of $\mathbf{AI}(\mathbf{A}+\mathbf{B})^{\mathrm{T}}\mathbf{IB}$?

20. Suppose $\mathbf{A}$ and $\mathbf{B}$ are $n \times n$ matrices. Simplify
$$\left(\frac{1}{2}\mathbf{A} + \frac{1}{2}\mathbf{B}\right)(2\mathbf{A} - 2\mathbf{B}) - \mathbf{BA}$$

21. Find the sum of the first 100 terms of the sequence whose terms are given by $a_n = e^{-n}$, $n = 0, 1, 2, \ldots$

22. The sum of the first 12 terms of an arithmetic sequence is 156. What is the sum of the second through 11^{th} terms?

Longer Problems

1. Let $f(x) = x^{100}$. What is the result of first expanding and simplifying $\dfrac{f(x+h) - f(x)}{h}$, and then setting $h = 0$? (Do not do the expansion— there is an easier way).

2. There are certain values of b for which the equation $3x^2 + bx + 3 = 0$ will have only one solution; what are they?

3. Find all values of θ in $[0, \pi/2]$ for which
$$2\sin^2\theta - 3\sin\theta + 1 = 0 \ .$$
Hint: let $x = \sin\theta$.

4. Graph $y = -3\cos(4x - \pi) + 2$ and state the period, amplitude and phase shift.

5. Graph $y = \frac{1}{2}\sin(2\pi x + \pi) - 1$ and state the period, amplitude and phase shift.

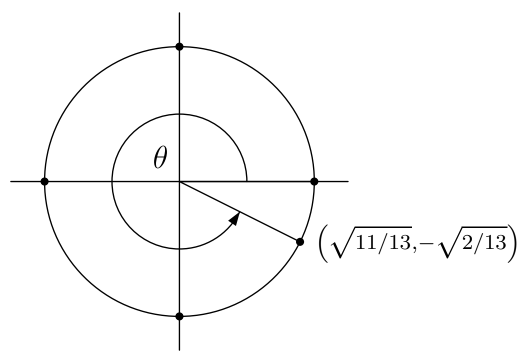
6. Consider the angle θ in the unit circle below:

   (a) Find $\cos\theta$.

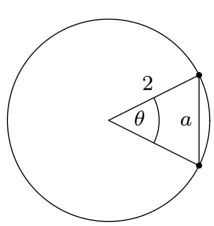
   (b) Find $\csc\theta$.

   (c) Find $\sin(\theta + \pi)$.

   (d) Find the coordinates of the point on the circle corresponding to the angle $\theta + \pi/2$.

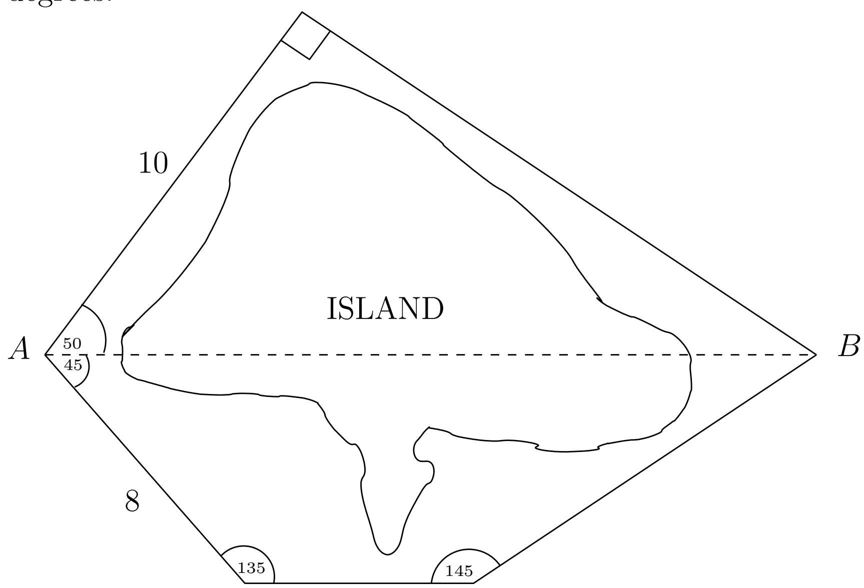


7. Find an expression for a in terms of θ using the figure below:

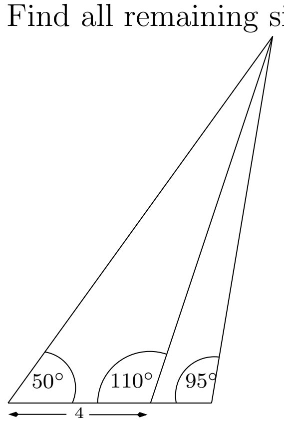


8. You measure the angle of elevation to the top of a mountain to be 40°. You then walk 300 metres directly away from the mountain and take another measurement, this time finding the angle of elevation to be 30°. How tall is the mountain?

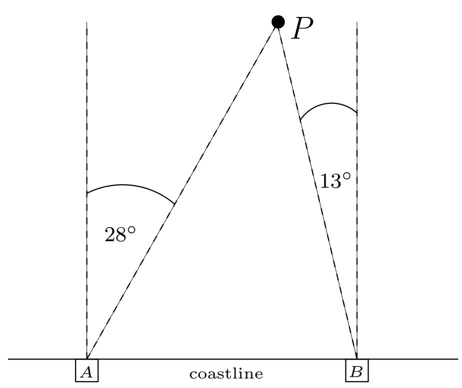
9. A ship at position A wishes to navigate around an island to point B using one of the two routes shown. Assuming the ship travels at the same constant speed over both routes, which should be chosen to complete the journey as quickly as possible? The angles shown are in degrees.



10. Find all remaining sides and angles in the following:



11. A plane makes an emergency landing in the ocean at point P. Rescue stations 9 kilometres apart at A and B both spot the plane and take the angle bearings shown. The rescue boat at A has a maximum speed of 40 knots, while that at B can do at most 35 knots. Which station should respond to the downed plane in order to reach the scene at quickly as possible?



12. Find the value of $\tan(\pi/8)$ exactly (trigonometric identities will help here.)

13. If $\sin a = 2/3$ and $\sin b = 1/7$, where both a and b are angles in $[0, \pi/2]$, compute and simplify $\cos(a-b)\cos(a+b)$.

14. Compute and simplify $\sin\left[\sin^{-1}(1/3) + \sin^{-1}(1/4)\right]$

15. Find the term containing x^{12} in the expansion of $\left(2x - \frac{1}{2x^2}\right)^{15}$.

16. Find the term containing a^8 in the expansion of $\left(a - \frac{2}{\sqrt{a}}\right)^{14}$.

17. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 2 \\ 1 & 1 & 3 \end{bmatrix}$$

   (a) Find $\mathbf{A^{-1}}$.

   (b) Use the inverse in part (a) to solve the system of equations $\mathbf{AX} = \mathbf{Y}$ where $\mathbf{X}^{\mathrm{T}} = \begin{bmatrix} x_{11} & x_{21} & x_{31} \end{bmatrix}$ and $y_{11} = 1$, $y_{21} = 4$, $y_{31} = 1$.

18. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -3 \\ 3 & -1 & -2 \\ 2 & 3 & -5 \end{bmatrix}$$

Suppose also that $\mathbf{B}$ is the matrix of size 3×1 whose entries are $b_{11} = -2$, $b_{21} = 1$ and $b_{31} = -3$.

   (a) Find the inverse of $\mathbf{A}$ or show that it does not exist.

   (b) Find all solutions to $\mathbf{AX} = \mathbf{B}$ where $\mathbf{X}^{\mathrm{T}} = \begin{bmatrix} x & y & z \end{bmatrix}$.

19. Let k be a constant and consider the system of linear equations

$$\begin{aligned} 2x - y + 3z &= -2 \\ x + y + 2z &= 4 \\ 3x - 3y + 4z &= k \end{aligned}$$

   (a) For what value of k does the system of equations have solutions?

   (b) For the value of k in part (a) find the solution corresponding to $z = 1$.

20. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 4 & -6 \\ -2 & 3 \end{bmatrix}.$$

   (a) Compute and simplify $(\mathbf{A} - 2\mathbf{I})(\mathbf{B} + 3\mathbf{I})$.

   (b) Find the inverse of the matrix from part (a) if possible.

21. The sum of three consecutive terms $x - d, x, x + d$ of an arithmetic sequence is 30, while their product is 360. Find the terms.

22. $\$P$ is deposited at the beginning of each year for 20 years into an investment paying 7% compounded annually. At the end of 20 years, the accumulated value is S_1. Deposits of $\$P$ are also made at the beginning of each year for 20 years into a second investment paying 8% compounded annually, and at the end of the 20 years the second investment has accumulated to a value of S_2. What is S_2/S_1?