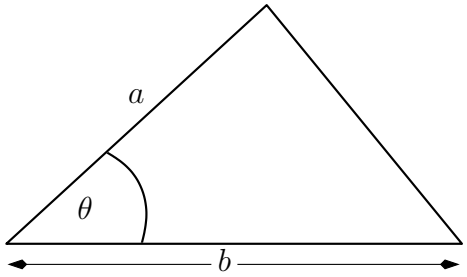
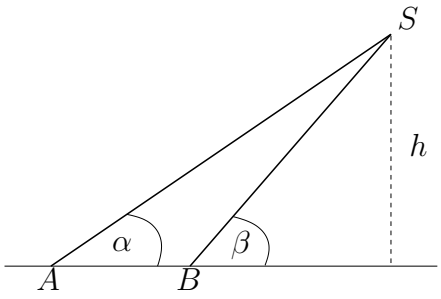
Problems

1. Recall that the area of a triangle is given as one half the base times the height: $A = \dfrac{bh}{2}$. With reference to the triangle below (not necessarily a right triangle), find a formula for the area A which is in terms of θ, a and b.



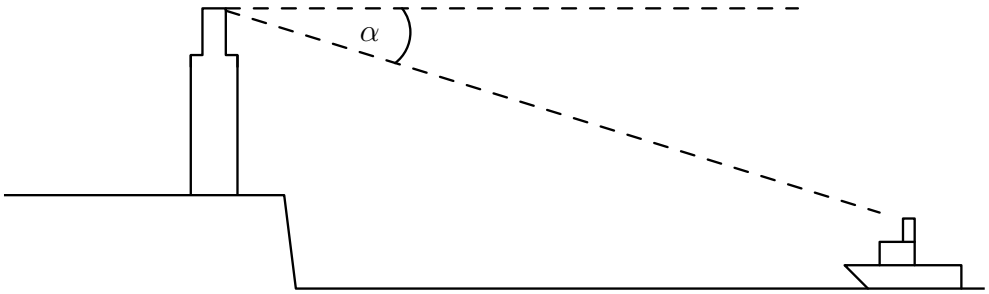
ans: $A = (1/2)ab\sin\theta$

2. Satellite tracking stations are located 200 km apart at positions A and B as shown in the figure below. A satellite in orbit passes directly over A and then B, and at a certain point in time later the stations measure the angles of elevation to the satellite at S as α and β as shown. Express the altitude h of the satellite in terms of the angles α and β.



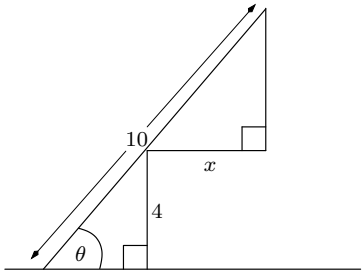
ans: $h = 200/(\cot\alpha - \cot\beta)$

3. A lighthouse keeper is standing on the lighthouse observation deck 25 m above the surface of the water. He observes a ship at an angle of depression α as shown in the figure below. If $\alpha = 0.7°$, approximately how far away is the ship? Round your answer to the nearest tenth of a kilometre.



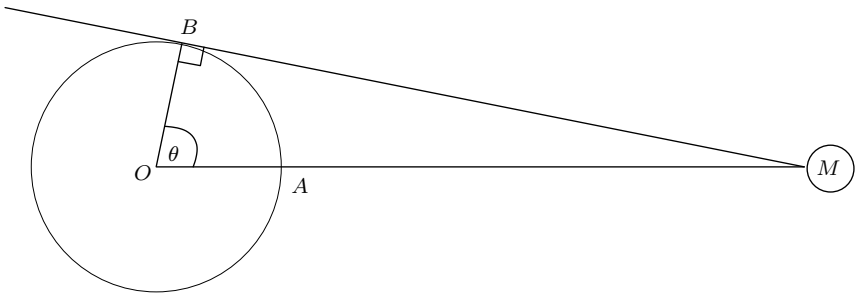
ans: 2.0 km

4. Referring to the figure below, find an expression for x in terms of θ.



ans: $x = 2(5\cos\theta - 2\cot\theta)$

5. At a certain point in time the moon is directly above point A on the earth, while at the same time it is just visible on the horizon from point B (see the figure below). Given that the shortest distance between A and B on the earth's surface is 9896 kilometres, and the circumference of the earth is 40,000 kilometres, what is the approximate distance from point A to the moon? Round to the nearest one thousand kilometres. (Hint: find angle θ first).



ans: 383,000 km

6. Compute the values of the following:

   (a) $\sec\left(\dfrac{113\pi}{6}\right)$.

   (b) $\cot\left(\dfrac{-50\pi}{4}\right)$.

   (c) $\sin\left(\dfrac{47\pi}{2}\right)\cos\left(-41\pi\right)\cot\left(\dfrac{59\pi}{2}\right)$.

   (d) $\sec\left(\pi\csc\left(13\pi/6\right)\right)$

ans: (a) $-2/\sqrt{3}$; (b) 0; (c) 0; (d) 1

7. Find all angles θ for which $\sin\theta = \csc\theta$.

ans: $\theta = (2k+1)\pi/2$ where k is any integer (i.e., any odd multiple of $\pi/2$)

8. Two cars leave the same location at 12:00 pm. One travels east along a straight road at 100 km/hr, while the other travels northwest at 110 km/hr along another straight road which makes an angle of 120° with the first road. At what time will the two cars be 400 km apart? (Round your answer to the nearest minute).

ans: 2:12 pm