Test 2 will cover sections 2.2 through 2.6 and 3.1 through 3.4. You should be familiar with all homework problems assigned in Asn 4, 5, 6, 7 & 8 as well as the theory covered up to and including Section 3.4 of the text.

As with Test 1, you will not be expected to produce long, original proofs of challenging, never before seen propositions. I may, however, ask you to give a short proof or two of propositions that are new to you but which I consider basic, or a proof of one of the basic yet important results from class (see Theorems and Proofs section below). You may also be asked to prove a result (or variation thereof) taken from the homework problems, or I may ask you to explain some aspect of one of the longer proofs we worked through in class. In addition to the homework material, you should be familiar with the material outlined below.

Definitions and Concepts

Be able to

1. State the definitions of $\limsup_{n\to\infty} x_n$ and $\liminf_{n\to\infty} x_n$ (Definition 2.3.1).
2. Determine, with explanation, the lim sup and lim inf of a given sequence: see Example 2.3.3.
3. State the Bolzano-Weierstrass theorem (Theorem 2.3.8).
4. State the definition of a Cauchy sequence (Definition 2.4.1).
5. State the definition of a series (Definition 2.5.1) and what is means for the series to converge.
6. State the definition of absolute convergence (Definition 2.5.14).
7. State the definition of the limit of a function (Definition 3.1.3).
8. Use the $\epsilon\,\delta$ definition of a limit to prove particular limit results: see Example 3.1.5.
9. Use sequential limits (Lemma 3.1.7) to prove limit results (Corollary 3.1.12 for example).
10. State the definition of a continuous function (Definition 3.2.1).
11. As with limits, use both the $\epsilon\,\delta$ definition of continuity and sequential limits to prove continuity (or discontinuity) of functions. (See Examples 3.2.3 and 3.2.11).
12. State the definition of uniform continuity (Definition 3.4.1).
13. Apply the definition of uniform continuity to show that a particular function is uniformly continuous (see Example 3.4.3 and textbook Exercise 3.4.11.).

Theorems and Proofs

Know how to prove the following results:

1. Proposition 2.2.5 (i). We did 2.2.5.(iii) in class, but (i) is easier and more direct.
2. Exercise 2.3.5 (no proof required, an explanation is fine)
3. Proposition 2.4.4: A Cauchy sequence is bounded.
4. Proposition 2.5.9: $\sum x_n$ converges implies $x_n \longrightarrow 0$.
5. Proposition 2.5.15: absolute convergence of a series implies convergence.
6. Proposition 3.1.4: The limit of a function is unique (not done in class, but a nice, short test question.)
7. Proposition 3.2.7: compositions of continuous functions are continuous.
8. Lemma 3.3.1: A continuous function on a closed and bounded interval is bounded.