Note: Structure your proofs by stating the full "**Proposition:**" followed by "**Proof:**" as in class, and please start each new proof on a new page. Indent your argument where appropriate for readability and be careful with notation. Keep your work neat and tidy, use complete sentences and correct punctuation, and please staple your assignment. Do not use pages torn from a coil notebook, or if you do, please trim off the ragged edge.

As with any proof, clarity of presentation is as important as solving the problem. Strive to make your proofs clear, concise and precise. Feel free to use results from earlier problems in subsequent ones.

Finally, proofs for many of the problems in this course are likely available online if one looks hard enough, but the point of this course is to learn analysis, and in particular how to write analysis proofs. As such, please resist turning to the internet for help except as a last resort. Instead, discuss solutions among your peers or see me for hints.

Late Assignment Policy: Assignments handed in t days late will incur a penalty of 4^t %. Here t is rounded up to the nearest integer.

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1. Prove that $f : \mathbb{R} \to \mathbb{R} : x \mapsto x^3$ is continuous at $x = -1$ using the $\epsilon\delta$ definition of continuity.
2. Prove that $f : (0, \infty) \to \mathbb{R} : x \mapsto 1/x$ is continuous using the $\epsilon\delta$ definition of continuity.
3. Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuous and that $f(c) > 0$. Show that there is some $\alpha > 0$ such that $f(x) > 0$ for every $x \in (c - \alpha, c + \alpha)$.
4. Show that if $f : [a, b] \to \mathbb{R}$ is continuous then $f([a, b])$ is either a closed and bounded interval or a single real number.
5. Suppose that $g : (a, b) \to \mathbb{R}$ is differentiable and that $g' : (a, b) \to \mathbb{R}$ is bounded. Show that g must also be bounded. (Hint: Mean Value Theorem.)
6. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function with the property that there exists an $\alpha > 1$ such that $|f(x) - f(y)| \leq |x - y|^\alpha$ for every x and y. Show that f must be a constant function.