Note: Structure your proofs by stating the full "**Proposition:**" followed by "**Proof:**" as in class, and please start each new proof on a new page. Indent your argument where appropriate for readability and be careful with notation. Keep your work neat and tidy, use complete sentences and correct punctuation, and please staple your assignment. Do not use pages torn from a coil notebook, or if you do, please trim off the ragged edge.

As with any proof, clarity of presentation is as important as solving the problem. Strive to make your proofs clear, concise and precise. Feel free to use results from earlier problems in subsequent ones.

Finally, proofs for many of the problems in this course are likely available online if one looks hard enough, but the point of this course is to learn analysis, and in particular how to write analysis proofs. As such, please resist turning to the internet for help except as a last resort. Instead, discuss solutions among your peers or see me for hints.

Late Assignment Policy: Assignments handed in t days late will incur a penalty of 4^t %. Here t is rounded up to the nearest integer.

Note: As noted previously, assume all sequences (resp. series) are sequences (resp. series) of real numbers unless told otherwise.

1. We've focused on proving convergence of sequences (and series) but not so much on showing divergence. Recall, a sequence (or series) is divergent if it is not convergent. Prove the following:
   (a) $\{\sqrt{n}\}_{n=1}^{\infty}$ is divergent.
   (b) $\{(-1)^n\}_{n=1}^{\infty}$ is divergent.
2. Let $\sigma : \mathbb{N} \to \mathbb{N}$ be a bijection. For a given series $\sum_{k=1}^{\infty} x_k$, let $y_k = x_{\sigma(k)}$. The series $\sum_{k=1}^{\infty} y_k$ is said to be a **rearrangement** of $\sum_{k=1}^{\infty} x_k$. Suppose that $x_k \geq 0$ for every $k \in \mathbb{N}$ and that $\sum_{k=1}^{\infty} x_k$ converges to X. Show that $\sum_{k=1}^{\infty} y_k$ also converges to X.

   [Hint: The partial sums of $\sum_{k=1}^{\infty} y_k$ are bounded above by X (why?)]

   Note: This result is part of a more general theorem due to Dirichlet (1805-1859): If $\sum_{k=1}^{\infty} a_k$ converges absolutely then so does any rearrangement of the series and all rearrangements converge to the same sum.
3. If $\displaystyle\sum_{k=1}^{\infty} x_k$ converges absolutely prove that
$$\left|\sum_{k=1}^{\infty} x_k\right| \leq \sum_{k=1}^{\infty} |x_k| \; .$$
4. For this question use the $\epsilon\delta$ definition of the limit:
   (a) Determine the limit (with proof) or show that it does not exist: $\displaystyle\lim_{x\to 1} \frac{1}{x}$.
   (b) Determine the limit (with proof) or show that it does not exist: $\displaystyle\lim_{x\to 9} \sqrt{x}$.
   (c) Determine the limit (with proof) or show that it does not exist: $\displaystyle\lim_{x\to c} (x^2 + x + 1)$ where c is any real number.